MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS, ABSTRACTS OF PAPERS, AND

REPORTS OF THE PROCEEDINGS

OF THE SOCIETY

FROM NOVEMBER 1916 *TO NOVEMBER* 1917.

VOL. LXXVII.

PUBLISHED BY THE ROYAL ASTRONOMICAL SOCIETY,
BURLINGTON HOUSE, LONDON, W. I,
AND SOLD BY
WILLIAM WESLEY & SON, 28 ESSEX STREET,
STRAND, LONDON, W.C. 2.
1917.

PRINTED IN GREAT BRITAIN BY NEILL AND CO., LTD., EDINBURGH.

MONTHLY NOTICES

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ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVII. NOVEMBER 10, 1916. No. 1

Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

James Hartness, Springfield, Vermont, U.S.A.;
Mrs. Annie Scott Dill Maunder, 8 Maze Hill, Greenwich Park, London, S.E.; and
Reginald Lawson Waterfield, The Principal's House, Cheltenham,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

The Hon. Sir Charles Algernon Parsons, K.C.B., M.A., D.Sc., LL.D., F.R.S., 6 Windsor Terrace, Newcastle-on-Tyne (proposed by R. A. Sampson);
Rohani Thirunal Raja M. Rama Varma Raja, Prince of Mavelikara Palace, Assistant Director and Astronomer, H.H. The Maharaja's Observatory, Trivandrum, Travancore, S. India (proposed by Raja A. V. Jugga Rao); and
Manning Robertson, A.R.I.B.A., 50 Norfolk Square, London, W. (proposed by Major P. A. MacMahon).

One hundred and twelve presents were announced as having been received since the last meeting, including, amongst others:—
Royal Observatory, Cape of Good Hope, Annals, vol. xii.

part 4: Determination of the Elements of the orbits of Jupiter's satellites from photographs by Bryan Cookson, presented by the Observatory;

Cordoba, Observatorio Nacional, Resultados, vol. xxi.: Catalogo de las Zonas de Exploración, $-52°$ á $-62°$, presented by the Observatory;

H. Knox Shaw: Photographs of the Variable Nebula in Corona Australis (24 lantern slides), presented by the Helwân Observatory;

Two photographs of the turret of the White Tower, used by Flamsteed as an Observatory, presented by Mr. Charles ffoulkes;

Thirty-nine lantern slides, various subjects, presented by Professor Hale.

On Chandler's Period in the Latitude Variation.

By Oskar Backlund.

Mr. Witting in an interesting investigation on the motion of the pole has collected all the international latitude observations from 1890 to 1914, and arranged them according to x (meridian of Greenwich) and y (perpendicular to x) for every tenth of a year. It seemed to me that this rich material might afford some indications concerning the variation of Chandler's period and the amplitude. I therefore undertook to calculate θ, the mean motion in 0·1 year, the amplitude and the phase for every tenth of a year. In order to render the calculations as simple as possible, the first task was to work out appropriate formulæ.

1. Employing the notations of Mr. Witting, we suppose the latitude may be expressed by

$$P = P_0 + \kappa \cos(\theta t + c),$$

unit of $t = 0{\cdot}1$ year.

Let n be the number of equidistant values of P covering nearly one period, and $m+1$ the number of periods, then, summing the $(m+1)$ values of P for a given value of ν, we have

$$\sum_{\mu=0}^{\mu=m} P_{\mu,\nu} = S_\nu = (m+1)P_0 + \kappa R \cos(\theta\nu + \psi),$$

where

$$R = \frac{\sin\frac{m+1}{2}n\theta}{\sin\frac{n}{2}\theta}; \quad \psi = \frac{mn}{2}\theta + c,$$

in which ν may have the values $0, 1, \ldots, (n-1)$.

If we take $n = 12$, $m = 4$, the influence of short periods, especi-

ally of that of one year, is nearly excluded. The following formula is easily verified:

$$H_\nu = \frac{S_\nu - S_{\nu+1}}{S_{\nu+3} - S_{\nu+4}} = \frac{\sin\left(\frac{2\nu+1}{2}\theta + \psi\right)}{\sin\left(\frac{2\nu+1}{2}\theta + \psi + 3\theta\right)}.$$

As an approximate solution we assume $\theta_0 = 30°$. We have

$$\theta = \theta_0 + \Delta\theta.$$

We put

$$V_\nu = \frac{2\nu+1}{2}\theta + \psi,$$

and obtain then

$$H_\nu = \frac{\sin V_\nu}{\cos(V_\nu + 3\Delta\theta)},$$

or

$$\frac{1}{H_\nu} = \cot V_\nu \cos 3\Delta\theta - \sin 3\Delta\theta.$$

Each of these expressions furnishes the following three formulæ, which are easily verified, and in which higher powers of $\Delta\theta$ than the first are omitted. W_ν stands for arctan H_ν, and Q for

$$\frac{\sin\frac{(m+1)}{2}n\theta}{\sin\frac{n}{2}\theta}\sin\frac{\theta}{2}.$$

$$\theta = \frac{1}{36}\sum_{\nu=0}^{\nu=5}(W_{n-1-\nu} - W_\nu),$$

$$\kappa = \frac{1}{24}Q\sum_{\nu=0}^{\nu=11}\{(S_\nu - S_{\nu+1})\sin V_\nu + (S_{\nu+3} - S_{\nu+4})\cos V_\nu\},$$

$$\psi = \frac{1}{12}\sum_{\nu=0}^{\nu=5}(W_{n-1-\nu} + W_\nu) - 6\theta - 3\Delta\theta.$$

2. The procedure of the calculation is as follows: Commencing with 1890·8, the sum of the respective x and y values for 1890·8, 1892·0, 1893·2, 1894·4, 1895·6 gives S_0; S_1 is formed in the same manner, commencing with 1890·9, and so on. Every consecutive group contains, then, 11 constituents of the preceding one; only the 12th is a new one. Each such group gives a value of θ, κ, and ψ. The means (M_x, M_y) of 12 consecutive values of the x and y components respectively, thus found, are contained in the annexed table. Thus No. 1 of each table is the mean of 12 values derived from x and y corresponding to 1890·8 . . . 1891·9; No. 2 to 1892·0 . . . 1893·3, etc. It is not difficult to understand that the values corresponding to odd numbers have no common

constituents. The same remark holds for the even numbers. No. 16 is extrapolated for the use of mechanical quadrature. No observations were available for this number. A small error in the extrapolated value is of no consequence for the result.

3. The values of θ_x in the column M_x, and of θ_y in the column M_y, are well represented by

$$\theta_x = 30^\circ\cdot26 - 0^\circ\cdot43 \cos[18^\circ\cdot74(t - 1892\cdot0)] + 0^\circ\cdot09 \sin[18^\circ\cdot74(t - 1892\cdot0)]$$
$$\theta_y = 30^\circ\cdot20 - 0^\circ\cdot43 \cos[18^\circ\cdot74(t - 1892\cdot0)] - 0^\circ\cdot01 \sin[18^\circ\cdot74(t - 1892\cdot0)]$$

t in units of a year.

These agree within the limits of their probable errors. We therefore conclude that there is no appreciable systematical difference between θ_x and θ_y. Taking the mean of M_x and M_y, we find from the mean values

$$\theta = 30^\circ\cdot23 - 0^\circ\cdot43 \cos\{18^\circ\cdot74(t - 1892\cdot0) + 9^\circ\}.$$

In the computations I have employed

$$\theta = 30^\circ\cdot23 - 0^\circ\cdot44 \cos\{18^\circ\cdot74(t - 1892\cdot0) + 11^\circ\},$$

which in regard of the probable errors may be considered as identical with the preceding formula.

From the last formula we find for the period

$$C = 434^d\cdot7\ (1 + 0\cdot01455 \cos\{18^\circ\cdot74(t - 1892\cdot0) + 11^\circ\}),$$

where t is expressed in units of a year, which gives

$$C_{max} = 441^d\cdot0\ ;\ C_{min} = 428^d\cdot5.$$

In order to have an idea of the bearing of the expression for C we will test it on observations made before those which have been used in its deduction. From the observations made with the vertical circle in Pulkovo, MM. Ivanoff and Bonsdorff have determined C for 1844 and 1869, and Nyrén for 1887·2.

	Bonsdorff, 1844.	Ivanoff, 1869.	Nyrén, 1887.
Period observed . .	428 days.	436 days.	433 days.
Period by formula .	428 ,,	437 ,,	434 ,,
κ observed . . .	0″·090	0″·203	0″·184
κ by formula . .	...	0″·13 (Chandler)	0″·169*
κ_x formula . . .	...	0″·142	0″·171

p'' (the value of κ in 1845) is provisionally taken = 0″·090

Our formula represents the observations pretty well 45 years backward from 1890·8. It is therefore probable that it will also represent observations with the same exactitude 45 years forwards from 1908·8.

4. The deduction of a representative formula for κ is not so

* One value 0″·40 omitted.

easy as that for θ. The trouble lies in the fact that it is impossible to express κ by a trigonometrical series of only one argument, ωt. The Table II. evidences a gradual increase of κ, besides the fluctuations which can be expressed by a Fourier's Series of ωT_1. From the observations in Pulkovo, it seems certain that κ on an average increased from 1845, when it was very small, to 1907·6. The hypothesis that κ had a minimum 1845 and a maximum 1907·6 may serve for the deduction of an interpolation formula. According to this hypothesis, we assume

$$\kappa = F + G,$$

in which

$$F = a + b\cos(\sigma T_1);\ \sigma = 2^\circ\cdot 875;\ T_1 = t - 1907\cdot 6,$$

and

$$G = \sum_i \{\alpha \cos i\omega T_1 + \beta \sin i\omega T_1\}.$$

By means of the values of κ (Table II.), given in columns M_x and M_y, I obtained

$$F_x = 0''\cdot 080 + 0''\cdot 080\cos(\sigma T_1) + \Gamma,$$
$$F_y = 0''\cdot 089 + 0''\cdot 089\cos(\sigma T_1) + \Gamma,$$
$$\Gamma = +0\cdot 456p'' - 0\cdot 534p''\cos\sigma T_1,$$
$$G_x = G_y = +0''\cdot 036\cos\omega T_1.*$$

p'' signifies the value of κ in 1845 expressed in seconds.

These formulæ can, in view of the hypothesis concerning the maximum and minimum of F, claim only the character of interpolation formulæ for 1845 — 1907·6. While θ_x and θ_y are identical, F_x and F_y are decidedly different.

5. Having computed ψ, the phase c was obtained by means of

$$c = \psi - \frac{mn}{2}\theta = \psi - 24\theta;$$

or, since $24\theta_0 = 4\pi$,

$$c = \psi - 24\Delta\theta.$$

As is seen from the Table III., the single values do not agree very well. The columns M_x and M_y respectively give as mean values

$$c_x = 124^\circ;\ c_y = 214^\circ.$$

Hence

$$c_y - c_x = 90^\circ.$$

* The more precise formulæ are:

$$G_x = +0''\cdot 035\cos\omega T_1 + 0''\cdot 009\cos 2\omega T_1 + 0''\cdot 003\cos 3\omega T_1 - 0''\cdot 014\sin\omega T_1 - 0''\cdot 011\sin 2\omega T_1 - 0''\cdot 004\sin 3\omega T_1.$$

$$G_y = +0''\cdot 036\cos\omega T_1 + 0''\cdot 007\cos 2\omega T_1 + 0''\cdot 000\cos 3\omega T_1 - 0''\cdot 004\sin\omega T_1 - 0''\cdot 010\sin 2\omega T_1 - 0''\cdot 002\sin 3\omega T_1.$$

Different values of p'' between 0''·0 and 0''·1 will not appreciably affect the coefficients of these series.

We regard this result as an accidental agreement with the perpendicularity of x and y, as the probable error of $c_y - c_x$ is $\pm 3^\circ$.

TABLE I.

No.	θ_x.	No.	θ_x.		M_x.	No.	θ_y.	No.	θ_y.		M_y.
	°		°		°		°		°		°
1	30·0	2	29·7	I.	29·9	1	29·5	2	30·1	I.	29·8
3	29·7	4	30·1	II.	29·9	3	30·2	4	30·1	II.	30·1
5	30·3	6	30·6	III.	30·4	5	29·8	6	30·2	III.	30·0
7	30·6	8	30·6	IV.	30·6	7	30·3	8	30·4	IV.	30·4
9	30·6	10	30·8	V.	30·7	9	30·7	10	31·0	V.	30·8
11	30·6	12	30·6	VI.	30·6	11	30·6	12	30·4	VI.	30·5
13	30·2	14	29·9	VII.	30·1	13	30·5	14	30·0	VII.	30·2
15	30·0	16	29·9	VIII.	29·9	15	29.7	16	29·8	VIII.	29·8

TABLE II.

No.	κ_x.	No.	κ_x.		M_x.	No.	κ_y.	No.	κ_y.		M_y.
	"		"		"		"		"		"
1	0·146	2	0·123	I.	0·135	1	0·156	2	0·141	I.	0·148
3	·113	4	·118	II.	·116	3	·144	4	·146	II.	·145
5	·124	6	·125	III.	·125	5	·145	6	·139	III.	·142
7	·121	8	·128	IV.	·125	7	·141	8	·137	IV.	·139
9	·126	10	·138	V.	·132	9	·134	10	·145	V.	·140
11	·158	12	·186	VI.	·172	11	·162	12	·193	VI.	·177
13	·205	14	·218	VII.	·211	13	·214	14	·229	VII.	·222
15	·219	16	·205	VIII.	·212	15	·226	16	·241	VIII.	·234

TABLE III.

No.	c_x 1892·0.	No.	c_x 1892·0.		M_x 1892·0.	No.	c_y 1892·0.	No.	c_y 1892·0.	M_y 1892·0.
	°		°		°		°		°	°
1	81	2	92	I.	87	1	186	2	167	177
3	87	4	77	II.	82	3	165	4	174	170
5	74	6	71	III.	73	5	173	6	162	170
7	89	8	71	IV.	80	7	166	8	163	165
9	41	10	66	V.	68	9	153	10	151	153
11	43	12	72	VI.	73	11	145	12	169	156
13	53	14	88	VII.	85	13	168	14	183	176
15	53	16	75	VIII.	79	15	190	16	168	179

The Mean Parallax of Stars of different Magnitudes.
By Sir F. W. Dyson and W. G. Thackeray.

The proper motions of nearly 12,000 stars between the limits of declination $+24^{\circ}\cdot0$ and $+32^{\circ}\cdot0$ have been determined by comparison of Greenwich observations 1906–1915 with the Astronomische Gesellschaft Catalogues (Leiden, Cambridge, and Berlin B) and the earlier observations of Bessel and Lalande.

To the Greenwich observations in right ascension corrections for magnitude equation have been applied which were derived by comparison of the observations of different observers with the positions given in Kustner's Catalogue. An independent check on the freedom of Kustner's Catalogue from magnitude equation was obtained from observations with the altazimuth in the meridian, using the travelling-wire micrometer. The accordance was very satisfactory.

To the Greenwich declinations a correction of $-0''\cdot4$ was applied to make them agree with the declinations of Boss's Fundamental Catalogue. To the catalogues of the Astronomische Gesellschaft the corrections given by Boss were applied both to right ascensions and declinations. With these data preliminary proper motions were obtained, and the results used to obtain systematic corrections to the separate zones of Bessel and Lalande. Proper motions were then obtained from Greenwich–Bessel and from Greenwich–Lalande. The adopted proper motions were formed by giving a double weight to the Greenwich–A.G.C. determination as compared with Greenwich–Lalande and Greenwich–Bessel.

The resulting proper motions are thus systematically Greenwich–A.G.C. The difference of epoch is generally from 30 to 40 years. When the difference was less than 20 years no determination was made. The number of stars for which a comparison with Bessel was possible is approximately 7000, with an interval of about 80 years; and with Lalande about 4500, with an interval of approximately 110 years.

The stars were divided according to magnitude into four groups—

$$6^{m}\cdot0\text{–}7^{m}\cdot8,\quad 7^{m}\cdot9\text{–}8^{m}\cdot5,\quad 8^{m}\cdot6\text{–}8^{m}\cdot9,\quad 9^{m}\cdot0\text{–}\quad ,$$

the magnitudes being those of the A.G. Catalogues. The means of the proper motions were taken for each hour, and the results analysed for the direction and amount of the solar motion. The following stars were omitted on account of their very large proper motions:—

Star.	Mag.	R.A.	Dec.	Centennial Motion. R.A.	Centennial Motion. Dec.
		h m	°	s	″
C 1581	8·1	3 3	26	−1·31	−83·6
C 3695	7·3	6 58	29	+1·21	−82·3
L 3313	8·2	7 48	31	+5·63	−184·8
C 4285	7·5	7 55	29	−1 02	−117·9
L 3472	8·3	8 13	31	−2·27	−81·4
B 5072	9·0	14 22	24	+5·66	−110·5
L 6797	8·7	18 38	31	+0·61	−83·5
C 14250	8·6	23 41	29	+7·01	+2·2
C 14413	6·9	23 57	27	+6·22	−98·6

With these omissions the means for each hour are as follows (the units being centennial proper motions of $0^{s}\cdot01$ and $0''\cdot1$):—

Hour.	$6^{m}\cdot0$–$7^{m}\cdot8$. R.A.	Dec.	No.	$7^{m}\cdot9$–$8^{m}\cdot5$. R.A.	Dec.	No.	$8^{m}\cdot6$–$8^{m}\cdot9$. R.A.	Dec.	No.	$9^{m}\cdot0$– R.A.	Dec.	No.
h h												
0–1	+22	−21	83	+13	−8	123	+6	−14	142	−2	+1	139
1–2	+21	−37	61	+6	−13	144	−2	−13	133	−2	−5	102
2–3	+14	−31	72	+6	−12	108	0	−10	111	+1	−9	99
3–4	+4	−30	72	+3	−28	128	+4	−21	113	−1	−12	93
4–5	+2	−25	62	+4	−13	89	+11	−22	89	+1	−17	92
5–6	−1	−23	109	−1	−13	184	+2	−15	208	−2	−11	144
6–7	−2	−22	104	+3	−20	198	+1	−15	265	−2	−11	163
7–8	−4	−25	69	+1	−26	125	−4	−20	196	−6	−12	121
8–9	−15	−44	67	−5	−24	127	−4	−24	128	−12	−16	96
9–10	−16	−48	56	−11	−20	112	−10	−16	115	−15	−12	88
10–11	−12	−25	62	−18	−33	104	−19	−23	129	−15	−19	64
11–12	−17	−21	68	−17	−21	72	−24	−13	86	−20	−10	75
12–13	−22	−24	76	−12	−21	90	−15	−20	113	−7	−8	59
13–14	−29	−16	61	−20	−12	97	−23	−3	84	−23	−6	74
14–15	−14	−18	56	−19	−9	99	−18	0	100	−11	−2	84
15–16	−10	−15	44	−18	−18	115	−12	−3	100	−16	+3	86
16–17	−13	−18	64	−2	−18	105	−5	−8	120	−11	−9	82
17–18	0	−24	78	−7	−1	148	−6	−9	164	−4	−13	140
18–19	+6	−2	102	+4	−5	187	+1	−8	196	−1	+3	200
19–20	+10	−9	165	+4	−1	229	+2	−2	309	+1	+1	269
20–21	+8	−4	111	0	−4	256	−2	−1	278	−1	+8	204
21–22	+11	−11	88	+2	−3	195	+3	+4	211	−2	+4	169
22–23	+4	−11	85	−1	−8	151	−2	−1	190	−9	+5	148
23–0	+20	−8	85	+2	−8	135	+7	−3	154	+1	0	130
			1900			3321			3734			2921

The results for each hour were taken to one more figure in the actual computations.

From the analysis of these figures giving equal weight to each hour we find :—

Analysis of the Proper Motions in Right Ascension.

m m	s	s	s
6·0–7·8	− ·017	+ ·002 sin α	+ ·182 cos α + 2″·40
7·9–8·5	− ·034	+ ·025 sin α	+ ^{s}·113 cos α + 1″·49
8·6–8·9	− ·044	+ ·023 sin α	+ ^{s}·100 cos α + 1″·32
9·0	− ·066	+ ·010 sin α	+ ^{s}·077 cos α + 1″·01

Analysis of the Proper Motions in Declination.

m m	″	″	″
6·0–7·8	− 2·13	− 1·09 sin α	+ 0·48 cos α
7·9–8·5	− 1·41	− 0·75 sin α	+ 0·65 cos α
8·6–8·9	− 1·08	− 0·86 sin α	+ 0·49 cos α
9·0	− 0·62	− 0·75 sin α	+ 0·49 cos α

These expressions of the mean proper motions are considered as arising from—

(1) Residual error in the observations from magnitude equation.
(2) Corrections to Newcomb's precession.
(3) Solar motion.

Corrections Δm and Δn to the precession will give rise to—

$\Delta m + \Delta n \tan \delta_0 \sin \alpha$ in the proper motions in right ascension,

and

$\Delta n \cos \alpha$ in the proper motions in declination,

where $\delta_0 = 28°$, the mean declination of the zone.

Let X, Y, Z be the components of the solar motion, so that

$$X = M \cos A \sin D, \quad Y = M \sin A \sin D, \quad Z = M \cos D,$$

where M = centennial parallactic motion,
A = R.A. of solar apex,
D = Dec. of solar apex.

Then the solar motion gives rise to the terms

$X \sin \alpha - Y \cos \alpha$ in the proper motions in R.A.,

and

$X \cos \alpha \sin \delta_0 + Y \sin \alpha \sin \delta_0 - Z \cos \delta_0$ in the proper motions in Dec.

Magnitude Equation and Precession.

The absolute term in the right ascensions may in part indicate a correction Δm to the precession, but this quantity is so mixed up with magnitude equation that it seems impossible to determine m in this way unless the correction for magnitude equation for the two catalogues is known with accuracy. The progressive change in the value of the constant term shows that it arises in all probability from erroneous magnitude equation.

If the interval between the A.G. and Greenwich Catalogues be taken roughly as 33 years, then the adopted values of magnitude equation are only in error by $^{s}\cdot006$, $^{s}\cdot011$, $^{s}\cdot015$, $^{s}\cdot022$ for the successive magnitude groups, and as the absolute term in the right ascensions does not enter into the determination of the solar motion it need not be considered further.

The correction Δn to the precession enters into the sin α terms of the right ascensions and the cos α terms of the declinations.

From the R.A.'s: $X + \Delta n \tan \delta_0 = +''\cdot03, \quad +''\cdot33, \quad +''\cdot30, \quad +''\cdot13.$
From the Decs. : $X \sin \delta_0 + \Delta n = +''\cdot48, \quad +''\cdot65, \quad +''\cdot49, \quad +''\cdot49.$

As there is no sign of progression with the different magnitudes, it is clear that X is very nearly zero, but if the mean be taken, we find

$$X = -''\cdot10 \quad \text{and} \quad \Delta n = +''\cdot58.$$

This value of X, in conjunction with $-1''\cdot60$ for the mean value of Y found in the next paragraph, gives $266\frac{1}{2}^\circ$ as the right ascension of the solar apex. If, however, the position of the solar apex be assumed to be 270°, we find

$$\Delta n \tan \delta_0 = +''\cdot20 \quad \text{or} \quad \Delta n = +''\cdot38 \text{ from R.A.}$$
$$\text{and} \quad \Delta n = +''\cdot53 \text{ from Dec.}$$

Assuming it to be at $273\frac{1}{2}^\circ$, we find

$$\Delta n \tan \delta_0 = +''\cdot10 \quad \text{or} \quad \Delta n = +''\cdot19 \text{ from R.A.}$$
$$\text{and} \quad \Delta n = +''\cdot48 \text{ from Dec.}$$

A positive correction of $+''\cdot4$ or $+''\cdot5$ to Newcomb's precession is indicated, in the direction of Struve-Peter's value.

Solar Motion. Greenwich—A.G.C.

The right ascension of the solar apex is seen from the preceding paragraph to be nearly 270°. This is in close accordance with determinations made from stars situated all over the sky, as, *e.g.*, those based on the proper motions of Boss's Catalogue.

The values of the Y co-ordinate of the solar motion are as follows:—

		$6^m\cdot0$–$7^m\cdot8$.	$7^m\cdot9$–$8^m\cdot5$.	$8^m\cdot6$–$8^m\cdot9$.	$9^m\cdot0$–	
	Y	$= 2''\cdot40$	$1''\cdot49$	$1''\cdot32$	$1''\cdot01$	from R.A.
	Y sin δ_0	$= 1''\cdot09$	$0''\cdot75$	$0''\cdot86$	$0''\cdot75$	from Dec.
or	Y	$= 2''\cdot32$	$1''\cdot59$	$1''\cdot83$	$1''\cdot59$	from Dec.

Weighting these in the ratio 4 : 1

$$Y = 2''\cdot42 \quad 1''\cdot51 \quad 1''\cdot42 \quad 1''\cdot13$$

The values of Z found from the declinations are—

	$Z \cos \delta_0$	=	2″·13	1″·41	1″·08	0″·62
	Z	=	2·41	1·59	1·22	0·70
Thus	tan D	=	·98	1·05	·86	·62
and	D	=	45°	46°	41°	32°

The adopted values of the parallactic motion for the different magnitudes are—

Limits of Mag. (B.D.)	6m·0–7m·8.	7m·9–8m·5.	8m·6–8m·9.	9m·0–
Mean Mag. . .	7m·25	8m·25	8m·75	9m·0
M	3″·40	2″·19	1″·87	1″·33

or corrected for magnitude equation in Dec. (see Addendum)—

M	3″·28	2″·23	2″·21	1″·71

The different values of the declination are not to be regarded as indicating any real difference in the direction of the solar apex for bright and faint stars. A personality in the bisection of stars of magnitude 9·0 relatively to those of magnitude 7·0 of only 0″·1 in amount would have changed Z by 0″·3, or from 0″·62 to 0″·92, thus increasing D from 32° to 39°. This point is fully discussed at the end of this paper.

We have therefore, in all what follows, adopted for the position of the solar apex A = 270°, D = 37°·5.

As the zone crosses the Milky Way, and at 12h is about 90° distant from it, mean parallactic motions were found for each separate hour, but the results gave no clear evidence of any systematic difference in different parts of the sky.

Solar Motion. Greenwich 1890—*Groombridge.*

To obtain further material, the position of the solar apex for stars of different magnitudes was redetermined from the Groombridge observations, stars of large proper motion being included, except the following :—

Star.	Mag.	R.A.	Dec.	Centennial Motion. R.A.	Centennial Motion. Dec.
	m	h m	°	″	″
Gr. 34	8·3	0 10	43·2	+285	+ 38
μ Cass.	5·3	0 58	54·2	+337	+156
Gr. 1618	6·8	10 2	50·2	−134	+ 51
Gr. 1830	6·5	11 44	38·8	+398	−577

The results are given in the following table :—

Group.	Mean Mag.	D.	M.	No. of Stars.
m m	m	°	″	
5·0–5·9	5·50	37	4·57	488
6·0–6·9	6·50	35	3·85	1048
7·0–7·9	7·50	34	2·84	1274
8·0–8·9	8·25	47	2·72	825

Solar Motion. Greenwich—Carrington.

Similarly, the parallactic motions were deduced for stars of different magnitudes in Carrington's Catalogue by comparison with the Greenwich Astrographic observations. The results are given in the following table :—

Group.	Mean Mag. (B.D. scale).	No. of Stars.	M.
m m	m		″
–7·9	7·42	124	3·13
8·0–8·8	8·50	521	2·56
8·9–9·1	9·00	447	2·08
9·2–9·3	9·26	598	2·14
9·4	9·40	472	1·36
9·5	9·50	1530	1·29

Solar Motion. Abbadia–Albany, A.G.C.

A comparison similar to that made between the Greenwich Catalogue for 1910 and the A.G. Catalogues of Leiden, Cambridge, and Berlin B has also been made between the Abbadia Catalogue and that of Albany. The observations of the declinations both at Abbadia and Albany were made by the use of parallel lines. At Abbadia corrections for magnitude equation in declination were applied to reduce the observations of separate observers to a uniform system. A comparison with Kustner gives the following magnitude equation to the Abbadia Catalogue places :—

$6^m·0–6^m·9$.	$7^m·0–7^m·9$.	$8^m·0–8^m·4$.	$8^m·5–8^m·9$.	$9^m·0–$
$-″·09$	$-″·11$	$-″·12$	$″·00$	$+″·15$

These corrections applied to the mean proper motions derived from the comparison of the declinations of the Abbadia and Albany Catalogues arranged in order of magnitude still left outstanding differences of such a character that it seemed likely that the Albany Catalogue may be affected to some extent in the same way. It was therefore decided to use only the right ascensions in this paper.

Group.	No. of Stars.	Parallactic Motion in R.A.	M.
m m		″	″
6·0–7·8	994	2·94	3·70
7·9–8·5	1481	2·57	3·24
8·6–8·9	2570	1·65	2·08
9·0	1879	1·29	1·63

Collected Results for Parallactic Motion.

The assumed magnitudes of the stars are on the scale of the Bonn Durchmusterung, with which the catalogues of the Astronomische Gesellschaft are in substantial agreement. These have been reduced to the Harvard scale by use of the tables given in *Harvard Annals*, vol. 54.

In the following table the magnitude on the Harvard scale is given in the first column. The source of the proper motions, number of stars, and the parallactic motion are given in succeeding columns. The mean parallax is derived from the parallactic motion by division by 410; that is, adopting 19·5 km. per second as the velocity of the solar motion, or 4·1 × the earth's distance from the sun in one year.

The discordances from the formula of the next paragraph are given in the last column.

Mag. (Harvard).	Source of Proper Motions.	No. of Stars in Group.	M.	Mean Parallax.	Difference from Formula.
m					
5·5	Greenwich 1890—Groombridge	488	4·57	·0111	+·0020
6·5	,, ,,	1,048	3·85	·0094	+ 5
7·3	Greenwich 1910—A.G.C.	1,900	3·28	80	− 1
7·3	Abbadia—Albany	994	3·70	90	− 11
7·5	Greenwich 1890—Groombridge	1,274	2·84	69	+ 6
7·5	Greenwich 1900—Carrington	124	3·13	76	− 1
8·3	Greenwich 1890—Groombridge	825	2·72	66	− 6
8·4	Greenwich 1910—A.G.C.	3,321	2·23	54	+ 5
8·4	Abbadia—Albany	1,481	3·24	79	− 20
8·5	Greenwich 1900—Carrington	521	2·56	63	− 6
9·0	Greenwich 1910—A.G.C.	3,734	2·21	54	− 4
9·0	Abbadia—Albany	2,570	2·08	51	− 1
9·3	Greenwich 1900—Carrington	447	2·08	51	− 5
9·4	Greenwich 1910—A.G.C.	2,921	1·71	42	+ 2
9·4	Abbadia—Albany	1,829	1·63	40	+ 4
9·7	Greenwich 1900—Carrington	598	2·14	52	− 11
10·0	,, ,,	472	1·36	33	+ 5
10·3	,, ,,	1,530	1·29	·0031	+ 4
		26,077			

The mean parallaxes are represented fairly satisfactorily by the formula

$$\log \varpi = -1{\cdot}224 - {\cdot}120\, m,$$

the differences in the sense formula − observation being shown in the last column.

The formula given by Kapteyn * is

$$\varpi = \text{''}\cdot 0135\ (0\cdot 78)^{m-5\cdot}$$

$$\text{or} \quad \log \varpi = -1\text{''}\cdot 275 - \cdot 108\ m.$$

Mag.	Mean Parallax.	
	Greenwich.	Kapteyn.
m	″	″
5·0	·0150	·0153
5·5	·0131	·0135
6·0	·0114	·0120
6·5	·0099	·0105
7·0	86	·0093
7·5	75	82
8·0	65	73
8·5	57	64
9·0	50	57
9·5	43	50
10·0	38	44
10·5	·0033	·0039

The agreement with Kapteyn's results lends no support to the view that the diminution of mean parallax with increasing magnitude is slower than is represented by his formula.

Addendum.

By adopting the position of the solar apex and the table of mean parallax, we are now enabled to compare the computed and observed values of Y and Z, and we get—

	$6^m\cdot0$–$7^m\cdot8$.	$7^m\cdot9$–$8^m\cdot5$.	$8^m\cdot6$–$8^m\cdot9$.	$9^m\cdot0$–	Mean.
Y (C − O)	+″·21	+″·41	+″·22	+″·32	+″·29
Z (C − O)	−·32	−·06	·00	+·31	+·02

Taking 33 years as the interval between the catalogues, the correction to Y will be produced by a correction to the right ascensions of $^s\cdot010 \cos\alpha$ in a direction to increase the solar motion.

The correction to Z evidently points to a magnitude equation in declination.

The declinations of the Cambridge Catalogue were made by bisecting the star with parallel lines 11″·2 apart, the same method as is used at Albany and Abbadia, and one which appears to be especially liable to introduce magnitude equation.

The system of corrections in Boss's Preliminary Catalogue were adopted as being the result of the latest material for forming

* Using 19·5 km./sec. for the solar motion.

such a system, and Boss has given no material for such a correction in declination, but Auwers has done so.

The quantities, less a constant given by Auwers in his tables for reducing star catalogues to his fundamental system, are as follows:—

Magnitude.	Cambridge.		Leiden.		Berlin B.	
	+	−	+	−	+	−
m	″	″	″	″	″	″
6·0		·13	·06			·02
6·5		·11	·04			·03
7·0		·09	·01			·02
7·5		·05		·01		·02
8·0		·00		·03		·01
8·5	·12			·03	·02	
9·0	·24			·03	·06	

A comparison between Kustner's Catalogue for 1900 and the Greenwich Catalogue for 1910 gives the following corrections to Greenwich declinations:—

$6^m·0-7^m·8$.	$7^m·9-8^m·5$.	$8^m·6-8^m·9$.	$9^m·0-$
+″·04	+″·03	−″·05	−″·06

Giving a weight of 5 to the Cambridge values, 2 to Leiden values, and 1 to Berlin B values, because Cambridge covers $\frac{5}{8}$, Leiden $\frac{2}{8}$, and Berlin B $\frac{1}{8}$ of the Catalogue zone, we get—

Limits of Mag.	Cambridge.		Leiden.		Berlin B.		Mean.		Greenwich.		Greenwich − A.G.C.	
	+	−	+	−	+	−	+	−	+	−	+	−
m m	″	″	″	″	″	″	″	″	″	″	″	″
6·0–7·8		·07$_5$		·00$_2$		·02$_1$		·04		·04$_1$		·08
7·9–8·5	·06$_5$		·03$_2$		·00$_1$		·03		·03		·00	
8·6–8·9	·18$_5$		·03$_2$		·04$_1$		·11			·05		·16
9·0–	·24$_5$		·03$_2$		·06$_1$		·15			·06		·21

In estimating the effect of these corrections on the proper motions it is to be remembered that the comparisons with Lalande and Bessel have to some extent reduced the effect of the magnitude equation of the A.G. Catalogues on the determined proper motions. This has been roughly allowed for by taking 40 years instead of 33 years as the difference of epoch.

Limits of Mag.	Correction to Centennial P.M.		$Z \cos \delta_0$.	$Z \cos \delta_0$. Corrected.	Z. Corrected.	Z (C − O).	
	+	−				+	−
m m	″	″	″	″	″	″	
6·0–7·8		·20	2·13	1·91	2·19	·23	
7·9–8·5		·00	1·41	1·41	1·59	·15	
8·6–8·9	·40		1·08	1·48	1·68	·45	
9·0–	·52		·62	1·14	1·27	·17	
						·25	

The correction to Z has now become a constant, and may be expressed as a correction to the observed declination of ″·09 in a direction to diminish the value of Z. As the corrections to the right ascensions are in the opposite direction, these corrections to Y and Z only slightly affect M, but materially alter D. The original values of M and as corrected for (1) magnitude equation and further corrected for (2) +″29 to Y and −″25 to Z are

	6m·0–7m·8.	7m·9–8m·5.	8m·6–8m·9.	9m·0–
M	3″·40	2″·19	1″·87	1″·33
(1)	3·28	2·23	2·21	1·71
(2)	3·35	2·26	2·24	1·78

On the Radiative Equilibrium of the Stars.
By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor.

1. *Outline of the Investigation.* — The theory of radiative equilibrium of a star's atmosphere was given by K. Schwarzschild in 1906.* He did not apply the theory to the interior of a star; but the necessary extension of the formulæ (taking account of the curvature of the layers of equal temperature) is not difficult. It is found that the resulting distribution of temperature and density in the interior follows a rather simple law.

Taking a star—a "giant" star of low density, so that the laws of a perfect gas are strictly applicable—and calculating from its mass and mean density the numerical values of the temperature, we find that the temperature gradient is so great that there ought to be an outward flow of heat many million times greater than observation indicates. This contradiction is not peculiar to the radiative hypothesis; a high temperature in the interior is necessary in order that the density may have a low mean value notwithstanding the enormous pressure due to the weight of the column of material above.

There is a way out of the difficulty, however, if we are ready to admit that the radiation-pressure due to the outward flow of heat may under calculable conditions of temperature, density, and absorption nearly neutralise the weight of the column, and so reduce the pressure which would otherwise exist in the interior. For the giant stars it is necessary that only a small fraction of the weight should remain uncompensated. (For the dwarf stars, on the other hand, radiation-pressure is practically negligible.)

We thus arrive at the theory that a rarefied gaseous star adjusts itself into a state of equilibrium such that the radiation-pressure very approximately balances gravity at interior points. This condition leads to a relation between mass and density on the one side and effective temperature on the other side, which seems to correspond roughly with observation. The laws arrived at differ considerably from those of Lane and Ritter.

* *Göttingen Nachrichten,* 1906, p. 41.

The principal results are given in §§ 7–10. The theory enables us to estimate the average densities of the giant stars of different spectral types; it shows that the average luminosity will be roughly the same for the different types, and determines this luminosity as compared with the Sun; it determines the maximum effective temperature which a star can attain; and it indicates the extent to which the masses of individual stars are likely to deviate from the mean mass. It is scarcely necessary to say that the conclusions here given are tentative, being based on analysis which is only concerned with obtaining a probable approximation; but there seems to be a satisfactory accordance with observation, so far as is known. The present results also remove an objection which might be urged against the theories of Lane and Ritter, viz. that they require the heat-energy retained in the star to be much greater than that generated by contraction.

The outermost layers of the star are outside the scope of this investigation, and the formulæ here given do not apply to them. In speaking of conditions at the boundary of the star, I refer to a depth negligible compared with the radius, but deep from the point of view of the spectroscopist. Hence the theory has no bearing on the interpretation of spectroscopic results. Frequent reference is made to the effective temperature, because it affords a measure of the total outflow of radiation; we can use this measure without discussing the conditions of the layer which actually possesses the effective temperature.

It is clear that we cannot arrive at much certainty with regard to the conditions in a star's interior, except in so far as the treatment can be based on the most general laws of nature. There are some physical laws so fundamental that we need not hesitate to apply them even to the most extreme conditions; for instance, the density of radiation varies as the fourth power of the temperature, the emissive and absorbing powers of a substance are equal, the pressure of a gas of given density varies as its temperature, the radiation-pressure is determined by the conservation of momentum, —these provide a solid foundation for discussion. The weak link in the present investigation is that I have assumed without much justification that a certain product $k\epsilon$ is constant throughout a star. I have given some evidence that if it is variable the general character of the results would not be greatly altered; and, as a step towards the elucidation of the problem of stellar temperatures, I plead to be allowed provisionally one rather artificial assumption.

2. *Radiative Equilibrium in the case of Spherical Symmetry.*— Schwarzschild's treatment deals with the case when the surfaces of equal temperature are parallel planes. We have to consider the case when they are concentric spheres. The fundamental principle is that the transfer of energy takes place by radiation; convection and conduction are considered negligible.

Let ξ be the distance of a point P from the centre O of a star. At P, let the intensity of radiation travelling outwards in a direc-

tion making an angle θ with OP be expressed as a series of zonal harmonics,

$$(A + B\cos\theta + CP_2(\cos\theta) + DP_3(\cos\theta) + \ldots)dS d\omega,$$

where dS is the cross-section of the stream of radiation, and $d\omega$ is the solid angle containing directions near to θ.

Consider a small cylinder of matter of cross-section dS, and with axis ds in the direction θ. The fraction of the radiation absorbed in this cylinder will be proportional to the density ρ, and may be set equal to

$$k\rho ds,$$

where k is a coefficient of absorption and represents the absorption by a cylinder of unit *mass* and unit cross-section. Assuming that each molecule absorbs independently of the others, it will make no difference whether the cylinder is long and of low density, or short and of high density.

Hence the loss of the beam of radiation in traversing the cylinder is

$$k\rho ds(A + B\cos\theta + CP_2(\cos\theta) + \ldots)dS d\omega \quad . \quad . \quad (1)$$

The matter in the cylinder emits energy equally in all directions at a rate proportional to (1) the mass contained, (2) the fourth power of the temperature T, and (3) the specific emissive power of the substance. The last quantity may be set equal to k, since the absorbing and emissive powers of a substance are necessarily equal. Hence the energy emitted from the cylinder in directions included in $d\omega$ is

$$\mu k T^4 \rho ds dS d\omega \quad . \quad . \quad . \quad . \quad (2)$$

where μ is an absolute constant of nature, connecting the units of energy and temperature.

The loss (1) and gain (2) together make up the change of intensity of the radiation in traversing a length ds. Hence

$$\frac{d}{ds}(A + B\cos\theta + CP_2(\cos\theta) + \ldots)ds dS d\omega$$
$$= -k\rho(A + B\cos\theta + CP_2(\cos\theta) + \ldots)ds dS d\omega + k\rho\mu T^4 ds dS d\omega \quad (3)$$

Now
$$\frac{d}{ds} = \frac{d\xi}{ds}\frac{d}{d\xi} + \frac{d\theta}{ds}\frac{d}{d\theta}$$
$$= \cos\theta\frac{d}{d\xi} - \frac{\sin\theta}{\xi}\frac{d}{d\theta} \text{ by geometry.}$$

Hence, since A, B, C are functions of ξ only, (3) becomes

$$\cos\theta\frac{dA}{d\xi} + \cos^2\theta\frac{dB}{d\xi} + \cos\theta \cdot P_2\frac{dC}{d\xi} + \frac{\sin^2\theta}{\xi}B - \frac{\sin\theta}{\xi}\frac{dP_2}{d\theta}C + \ldots$$
$$= -k\rho(A + B\cos\theta + CP_2 + \ldots) + k\rho\mu T^4 \quad (4)$$

Let us choose a unit of length comparable with the radius of the star. Then k is a very large quantity, since radiation must be strongly absorbed in a length small compared with the radius. It is then found that A, B, C rapidly diminish, so that B is of order k^{-1} and C of order k^{-2} compared with A. We shall therefore neglect C on the left side of (4), retaining it on the right, where it is multiplied by the large factor k.

Arrange the left side in zonal harmonics by using

$$\cos^2\theta = \tfrac{1}{3} + \tfrac{2}{3}\mathrm{P}_2(\cos\theta),$$
$$\sin^2\theta = \tfrac{2}{3} - \tfrac{2}{3}\mathrm{P}_2(\cos\theta),$$

and equate coefficients.

The constant terms give

$$\tfrac{1}{3}\frac{d\mathrm{B}}{d\xi} + \tfrac{2}{3}\frac{\mathrm{B}}{\xi} = k\rho(-\mathrm{A} + \mu\mathrm{T}^4) \quad . \quad . \quad . \quad (5)$$

The coefficients of $\cos\theta$ give

$$\frac{d\mathrm{A}}{d\xi} = -k\rho\mathrm{B} \quad . \quad . \quad . \quad . \quad (6)$$

The coefficients of $\mathrm{P}_2(\cos\theta)$ give

$$\tfrac{2}{3}\frac{d\mathrm{B}}{d\xi} - \tfrac{2}{3}\frac{\mathrm{B}}{\xi} = -k\rho\mathrm{C},$$

which verifies the statement made above that C is small compared with B. We shall therefore neglect C henceforth.

Equation (5) can be written

$$\frac{1}{\xi^2}\frac{d}{d\xi}(\mathrm{B}\xi^2) = 3k\rho(-\mathrm{A} + \mu\mathrm{T}^4) \quad . \quad . \quad . \quad (7)$$

To interpret the quantity B, we remark that the total radiation flowing outwards across unit surface transverse to the radius is obtained by integrating $(\mathrm{A} + \mathrm{B}\cos\theta)\cos\theta d\omega$ over the corresponding hemisphere. The factor $\cos\theta$ is required, since the cross-section of a beam of radiation flowing obliquely through a surface $d\mathrm{S}$ is equal to $d\mathrm{S}\cos\theta$. The result is

$$\pi\mathrm{A} + \tfrac{2}{3}\pi\mathrm{B} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Similarly the amount flowing outwards is

$$\pi\mathrm{A} - \tfrac{2}{3}\pi\mathrm{B} \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

Thus the net flow outwards is $\frac{4}{3}\pi\mathrm{B}$ per unit area, or $\frac{16}{3}\pi^2\mathrm{B}\xi^2$ across the sphere of radius ξ.

In a strictly steady state the total energy between two boundaries must be constant, and therefore the net outward flow across all boundaries must be the same. Thus $\mathrm{B}\xi^2$ is constant, and by (7)

$$\mathrm{A} = \mu\mathrm{T}^4.$$

In an actual star the stream of energy flowing outwards is supplied by slow changes occurring within the star. The simplest theory results if we suppose that the energy is produced by radioactive processes. Let the amount thus liberated per unit mass be $4\pi\epsilon$. Then in the steady state the outward flow of heat across the outer boundary of a spherical shell will exceed that across the inner boundary by the amount of heat generated in the shell. That is,

$$d(\tfrac{16}{3}\pi^2 B\xi^2) = 4\pi\epsilon \cdot 4\pi\rho\xi^2 d\xi.$$

Hence

$$\frac{1}{\xi^2}\frac{d}{d\xi}(B\xi^2) = 3\rho\epsilon \quad . \quad . \quad . \quad . \quad (9)$$

Imagine the successive spherical layers to be expanded or contracted until the whole star has a uniform density τ. Let x be the radius in the uniform star, which corresponds to ξ in the original star. Then

$$\rho\xi^2 d\xi = \tau x^2 dx \quad . \quad . \quad . \quad . \quad (10)$$

Hence from (6) and (9)

$$\frac{1}{\tau}\frac{dA}{dx} = -kB\frac{x^2}{\xi^2} \quad . \quad . \quad . \quad . \quad (11)$$

$$\frac{1}{\tau}\frac{d}{dx}(B\xi^2) = 3\epsilon x^2 \quad . \quad . \quad . \quad . \quad (12)$$

Integrating (12),

$$B = \tau\epsilon\frac{x^3}{\xi^2} \quad . \quad . \quad . \quad . \quad (13)$$

and

$$\frac{1}{\tau}\frac{dA}{dx} = -k\tau\epsilon\frac{x^5}{\xi^4} \quad . \quad . \quad . \quad . \quad (14)$$

If the outflowing energy is produced by contraction instead of by radioactivity, it is not so easy to give a precise statement, because, strictly speaking, the conditions are changing with the time. We may write down equation (13) as a definition of ϵ; and it will be seen that $4\pi\epsilon$ is then the net flow of radiation across the spherical surface divided by the mass within the surface, *i.e.* the average energy per unit mass generated by contraction. I shall generally take ϵ to be constant, representing roughly a state of affairs such that the energy of stellar radiation comes from processes going on in all parts of the star, and not from a singularity at the centre. It must be noticed that, except near the surface, ϵ is extremely small compared with μT^4, so that in any case $A = \mu T^4$ approximately.

Let g be the value of gravity at ξ

and G the constant of gravitation,

then $$g = \tfrac{4}{3}\pi G\tau x^3/\xi^2,$$

since the numerator gives the total mass within a sphere of radius ξ.

The pressure equation is

$$\frac{1}{\rho}\frac{dp}{d\xi} = -g,$$

whence by (10)

$$\frac{1}{\tau}\frac{dp}{dx} = -g\frac{x^2}{\xi^2}$$

$$= -\tfrac{4}{3}\pi G\tau\frac{x^5}{\xi^4}$$

$$= \tfrac{4}{3}\frac{\pi G}{k\epsilon}\frac{1}{\tau}\frac{dA}{dx} \quad \text{by (14)} \quad . \quad . \quad (15)$$

If k and ϵ are constants, integrating and setting $A = \mu T^4$,

$$p = \frac{4\pi\mu G}{3k\epsilon}T^4 \quad . \quad . \quad . \quad . \quad (16)$$

Strictly speaking, there is a small constant of integration, but it can be neglected in comparison with the large values of p in the interior of the star (see § 5 (*b*)).

Up to this point we have not used the gas-equation. For a *gaseous* star obeying Boyle's Law, $p = R T \rho$. Hence from (16), $\rho \propto T^3$ and $p \propto \rho^{\frac{4}{3}}$.

The distribution of density, temperature, and pressure for radiative equilibrium is identical with that occurring in the adiabatic equilibrium of a mass of gas for which the ratio of the specific heats is $\frac{4}{3}$.*

A gas for which $\gamma = \frac{4}{3}$ can be at the same time in radiative and adiabatic equilibrium; if γ alters, adiabatic equilibrium alters but radiative equilibrium does not.

3. *Intensity of the Outward Radiation.*—The net outward radiation has been found to be $\frac{16}{3}\pi^2 B\xi^2$

$$= \tfrac{16}{3}\pi^2\tau\epsilon x^3 \quad \text{by (13)}.$$

If the effective temperature of the star is T_1, the outflow is equivalent to that due to isotropic radiation of intensity $A_1 = \mu T_1^4$. By (8) this outflow is πA_1 per unit area. Hence

$$\pi\mu T_1^4 \cdot 4\pi\xi^2 = \tfrac{16}{3}\pi^2\tau\epsilon x^3 \quad . \quad . \quad . \quad (17)$$

where x and ξ have their surface values.

Eliminate ϵ between (16) and (17), we obtain

$$kT_1^4 = \tfrac{4}{3}\pi G\tau\frac{x^3}{\xi^2}\frac{T^4}{p}$$

$$= \tfrac{4}{3}g_1\frac{T^4}{p} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where g_1 is the value of gravity at the surface.

* It may be well to restate the restriction, viz. that $k\epsilon$ must be constant. This point is discussed in § 5.

This equation gives the relation between the constant of absorption and the effective temperature. T and p are any corresponding values of the temperature and pressure. It is usually most convenient to use the central values, T_0 and p_0, so that

$$k = \tfrac{4}{3}\frac{g_1}{p_0}\left(\frac{T_0}{T_1}\right)^4 \quad . \quad . \quad . \quad . \quad (19)$$

4. *Numerical Example.*—Before discussing possible modifications of the somewhat ideal conditions assumed, we shall give an example. The necessary tables and formulæ are given in Emden's *Gaskugeln* (pp. 85, 69). Emden actually works out this case (regarded as a case of adiabatic equilibrium) for the Sun, but we shall take a star in a more perfect gaseous condition. For the method of calculation reference may be made to Emden's examples (p. 96).*

Let the radius of the star be $r = 7 \times 10^{11}$ cm.
Let the mean density be $\rho_m = 0{\cdot}002$ gm. cm.$^{-3}$
Then the mass is $M = 2{\cdot}87 \times 10^{33}$ gm. (about $1{\cdot}5 \times$ Sun).

The central density in this form of equilibrium is *always* 54·2515 times the mean density. In this case

$$\text{Central density } \rho_0 = 0{\cdot}1085,$$

so that no serious deviation from Boyle's Law will occur.

To calculate the temperature and pressure, the only other quantity needed is the molecular weight. I have taken this to be 54 as a likely average (*e.g.* monatomic iron vapour). The corresponding value of R is $1{\cdot}536 \times 10^6$. I find at the centre

$$T_0 = 1{\cdot}52 \times 10^8 \text{ degrees Centigrade}$$
$$p_0 = 2{\cdot}535 \times 10^{13} \text{ dynes cm.}^{-2}$$

At the surface

$$g_1 = 390{\cdot}5 \text{ cm. sec.}^{-2}$$

Temperature gradient = 6°·35 per kilometre.

(The temperature gradient is nearly uniform for a great distance inwards; it increases slowly to a maximum of $6\frac{1}{2}$ times its surface value at a depth equal to $\frac{3}{4}$th of the radius.)

The following formulæ show how the conditions at the centre are altered by varying the mean density, mass, molecular weight (R^{-1}), or constant of gravitation:

$$\left.\begin{aligned} \rho_0 &\propto \rho_m \\ T_0 &\propto \rho_m^{\frac{1}{3}} M^{\frac{2}{3}} G R^{-1} \\ p_0 &\propto \rho_m^{\frac{4}{3}} M^{\frac{2}{3}} G \end{aligned}\right\} . \quad . \quad . \quad . \quad (20)$$

To calculate k we must assume a value of the effective temperature. A star having the assumed mean density would probably be

* There is a misprint in the formula for Θ^3 on p. 97. The last bracket in the denominator should be squared.

of type F – G, and have an effective temperature of, say, 6500° absolute.

Substituting in (19) we find

$$k = 6{\cdot}2 \times 10^{6}.$$

This value is quite absurd, for it indicates that the radiation would be almost wholly absorbed in a path of 10^{-6} cm. at unit density, or 10^{-3} cm. at atmospheric density. Such a high opacity can scarcely be reconciled with the known properties of matter.

Inverting the argument and taking a more reasonable value of k, say 40, the effective temperature would have to be 130,000°C., which is, of course, out of the question.

A further indication that something is wrong is given by a calculation of the total amount of radiant energy contained in the star. The radiant energy per unit volume is aT^4 ergs, where $a = 7{\cdot}1 \times 10^{-15}$ and T is reckoned in degrees. On integrating throughout the volume, we find the total imprisoned radiant energy is

$$H = \tfrac{1}{2}\frac{aT_0^4 GM^2}{p_0 r} = 5{\cdot}85 \times 10^{52} \text{ ergs},$$

whereas the whole energy so far generated by contraction is

$$\Omega = \tfrac{3}{2}\frac{GM^2}{r} = 1{\cdot}18 \times 10^{48} \text{ ergs}.$$

One naturally asks, Where has all the radiant energy come from?

The radiant energy probably possesses electromagnetic mass; but this does not give rise to any difficulty. The mass is found by dividing H by the square of the velocity of light

$$H/c^2 = 6{\cdot}5 \times 10^{31} \text{ grams},$$

which is less than $\frac{1}{40}$th of the whole mass of the star.

5. *Discussion of the Assumptions.*—The last paragraph shows that the theory has led to improbable results. We therefore examine the assumptions made.

(*a*) *The Radiative Hypotheses.*—It can scarcely be doubted that at the high temperatures which (on any theory) are held to prevail in the star, radiation is far more effective than any other cause in transferring heat. Its importance rapidly increases as the temperature rises, since it depends on the fourth power of the temperature. Convection currents would only arise if the transfer of heat by radiation were not sufficiently rapid to prevent the outer parts from cooling and sinking; our difficulty is that the transfer by radiation seems to be much too rapid.

(*b*) *The Boundary Condition.*—It will be found that the terms A, B, C become of the same order of magnitude near the boundary and our equations break down. The stellar atmosphere terminates in a way different from what we have supposed, and the correct equations are those given by Schwarzschild. The boundary tem-

perature is, in fact, $T_1 / \sqrt[4]{2}$ instead of zero, and the pressure-temperature relation is more nearly

$$p \propto T^4(1 - T_1^4/2T^4);$$

that is to say, a small constant of integration is required in (16).

Until we are close to the boundary T_1^4 is very small compared with T^4, and it is easy to verify that as regards the interior of the star the correction is insignificant. Our calculation of k in § 3 is not affected by any failure of the equations at the boundary—we have not used T_1 as the temperature of a definite layer, but as measuring the total outflow of heat. The actual outflow must evidently be continuous with the outflow across a sphere contained a little within the boundary where our equations apply.

(*c*) *Variation of* k.—It has been assumed that k is constant, but it may vary with the temperature—either of the absorbing matter or of the radiation to be absorbed.

We can make calculations assuming that k varies as any power of the temperature, say T^δ. If $k = \kappa T^\delta$, then instead of (15) we have

$$\frac{dp}{dx} = \frac{4\pi G\mu}{3\kappa\epsilon} \cdot \frac{1}{T^\delta} \frac{dT^4}{dx} \quad . \quad . \quad . \quad . \quad (21)$$

whence $p \propto T^{4-\delta}$.

The equilibrium is still a form of adiabatic equilibrium, but with a different value of the ratio of the specific heats γ. Reference to the examples worked out by Emden shows that the central temperature is not very seriously altered by changing γ. We can show this best by proceeding at once to an extreme case. Let k vary as T^3, then $p \propto T$ and the star is a sphere of uniform density. I find in this case for the star discussed in § 4,

$$T_0 = 8{\cdot}9 \times 10^7 \text{ degrees Centigrade}$$
$$p_0 = 2{\cdot}7 \times 10^{11} \text{ dynes cm.}^{-2}$$

Temperature gradient near surface = 25°·4 per kilometre.

It is evident from the slight alteration of the central temperature that the outflow of radiation and store of radiant energy will again be excessively great. In fact, for an effective temperature of 6500° we find

$$k = 2{\cdot}7 \times 10^8 \text{ at the centre.}$$

(*d*) *Variation of* ϵ.—Similar small changes are produced if the radiated heat is not contributed by all parts equally, but comes more from the centre than the exterior layers, or vice versa. There are two cases in which we can give exact results:—

If $\epsilon \propto (1 - \xi^2/r^2)^3$, the density is constant.

If $\epsilon \propto (c^2 + \xi^2)$, where c is a certain constant, the case corresponds to adiabatic equilibrium with $\gamma = 1{\cdot}2$.*

* ϵ is defined by equation (13).

The two cases correspond to the heat being generated mainly in the interior and mainly in the exterior parts respectively, and give some idea of the range that is possible.

(*e*) *Effect of Scattering.*—In the introduction to his theory of radiative equilibrium Schwarzschild states that he has neglected *scattering*; but it appears that scattering can be included—at least in the simple theory given by him. He considers radiation of intensity A propagated along the positive direction of ξ, and radiation of intensity B along the negative direction. The equations, taking account of absorption and emission, are then

$$\left.\begin{aligned} \frac{dA}{d\xi} &= -k\rho A + k\rho\mu T^4 \\ -\frac{dB}{d\xi} &= -k\rho B + k\rho\mu T^4 \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

If there is a coefficient of scattering s, so that, considering the radiation A, in a length $d\xi$, $\frac{1}{2}s\rho d\xi A$ is scattered forwards and the same amount backwards, the equations become

$$\left.\begin{aligned} \frac{dA}{d\xi} &= -k\rho A + k\rho\mu T^4 - \tfrac{1}{2}s\rho A + \tfrac{1}{2}s\rho B \\ -\frac{dB}{d\xi} &= -k\rho B + k\rho\mu T^4 - \tfrac{1}{2}s\rho B + \tfrac{1}{2}s\rho A \end{aligned}\right\} \quad . \quad (22a)$$

whence

$$\frac{d}{d\xi}(A - B) = k\rho(2\mu T^4 - A - B)$$

$$= 0 \text{ for the steady state}$$

$$\frac{d}{d\xi}(A + B) = -(k+s)\rho(A - B).$$

These differ only from the corresponding equations derived from (22) by the replacement of k by $(k+s)$. Thus scattering should be included in the absorption coefficient, and the theory then applies without modification.

6. *Radiation-Pressure.*—It appears then that no modification of the assumptions mentioned in § 5 could affect the main result, viz. that the interior temperature is so high as to cause much too great an outflow of radiation, unless we imagine the material to be almost perfectly opaque. The high temperature in the interior is inevitable if the gas—necessarily of moderate density in a giant star—is to support the weight of the enormous column of material above.

This points to a way out of the discrepancy—the weight of the column may be partly supported by radiation-pressure. In introducing radiation-pressure at this stage we do so not as a *hypothesis* to explain the discrepancy, but because in the conditions we have found radiation-pressure would be extremely powerful. Whether radiation-pressure is important or not depends on the value of k. If we had arrived at a low value

of k, our neglect of radiation-pressure would have been justified. With the high value found radiation-pressure would far outbalance gravity, and our neglect of it is clearly illegitimate.

As there seems to be a rather widespread impression that gases are not subject to radiation-pressure, it may be advisable to state the theory briefly. The pressure is simply a consequence of absorption or scattering. A beam of radiation carries a certain forward-momentum proportional to its intensity; after passing through a sheet of absorbing medium a weaker beam emerges carrying proportionately less momentum; the difference of incident and emergent momentum is retained by the medium and constitutes the pressure. The medium, in fact, absorbs the momentum of the beam in the same proportion as it absorbs the energy. The calculations of radiation-pressure on small solid particles are simply calculations of absorption and scattering by these particles; it is not possible to apply such methods to atoms and molecules, which absorb by some internal mechanism. But the relation between absorption and pressure is a perfectly general one, depending only on the conservation of momentum.

Consider a small disc of thickness $d\xi$, and let radiation carrying momentum h fall on it, travelling at an angle θ to the normal. The length of path is $d\xi \sec\theta$. Hence the momentum absorbed is $k\rho h d\xi \sec\theta$. Resolving along the normal, the normal outward momentum absorbed is $k\rho h d\xi$, which is independent of θ.

In the present problem we have energy $\pi A + \frac{2}{3}\pi B$ flowing outwards and $\pi A - \frac{2}{3}\pi B$ inwards across unit surface; hence the net outward momentum absorbed is $c \cdot \frac{4}{3}\pi B k\rho d\xi$, where c is the factor relating the momentum and energy of a beam. Now the pressure on a black body is numerically equal to the density of the energy (assumed isotropic). The density of the energy at temperature T is aT^4, where a is the universal constant $7{\cdot}06 \times 10^{-15}$. The outward flow of isotropic energy across unit surface is $\pi A = \pi\mu T^4$, and the outward flow of momentum is $c \cdot \pi\mu T^4$. Hence

$$aT^4 = c\pi\mu T^4,$$

so that

$$c = a/\pi\mu.$$

It follows that the force of radiation-pressure on an element $d\xi$ is

$$\tfrac{4}{3}\frac{a}{\mu} B k\rho d\xi \quad . \quad . \quad . \quad . \quad (23)$$

$$= -\tfrac{4}{3}\frac{a}{\mu} dA \quad \text{by (6)}$$

$$= -\tfrac{4}{3} a d(T^4) \, . \quad . \quad . \quad . \quad . \quad (24)$$

Equation (24) is quite general. Assuming now the constancy of $k\epsilon$ so that $T^4 \propto p$, the radiation-force is

$$-\tfrac{4}{3} a \frac{T_0^4}{p_0} dp.$$

For the star discussed the values of T_0 and p_0 in § 4 give

$$\tfrac{4}{3}a\frac{T_0^4}{p_0} = 2{\cdot}0 \times 10^5,$$

so that radiation-pressure would be 200,000 times stronger than gravitation. Of course, the result does not mean that the existing radiation-pressure is so strong as this ; it shows that our conclusions contradict our premises, viz. that radiation-pressure was negligible.

We must now form the modified equations in which radiation-pressure is included. The weight of an element $g\rho d\xi$ will be partly counterbalanced by the radiation-force (23)

$$\tfrac{4}{3}\frac{a}{\mu}\mathrm{B}k\rho d\xi.$$

Hence in the pressure-equation we replace g by

$$g - \tfrac{4}{3}\frac{a}{\mu}\mathrm{B}k.$$

Using (13), this gives

$$g - \tfrac{4}{3}\tau\frac{x^3}{\xi^2}\cdot\frac{ak\epsilon}{\mu}$$

$$= g\left(1 - \frac{ak\epsilon}{\pi \mathrm{G}\mu}\right) = \beta g, \quad \text{say} \quad . \quad . \quad . \quad (25)$$

We shall suppose as before that $k\epsilon$ is constant, and accordingly β is constant. Evidently the effect of radiation-pressure is exactly equivalent to an alteration of the constant of gravitation to βG. We see by (20) that the density distribution will be unaltered, but the pressures and temperatures previously calculated must be multiplied by β. Instead of (16) we shall have

$$p = \frac{4\pi\mu\beta \mathrm{G}}{3k\epsilon}\mathrm{T}^4 \quad . \quad . \quad . \quad . \quad (26)$$

To determine β we have from (25)

$$1 - \beta = \frac{ak\epsilon}{\pi \mathrm{G}\mu}$$

$$= \tfrac{4}{3}a\beta\frac{\mathrm{T}^4}{p} \quad \text{by (26)}, \quad . \quad . \quad . \quad (26a)$$

so that

$$1 - \beta = \tfrac{4}{3}a\beta^4\left(\frac{T_0^4}{p_0}\right)_c \quad . \quad . \quad . \quad . \quad (27)$$

where $(T_0^4/p_0)_c$ refers to the values calculated in § 4 without taking account of radiation-pressure.

Also k is given by changing g_1 to βg_1 in (19)

$$\tfrac{3}{4}k = \frac{\beta g_1}{T_1^4}\frac{T^4}{p},$$

or

$$\tfrac{3}{4}k = \beta^4\frac{g_1}{T_1^4}\left(\frac{T_0^4}{p_0}\right)_c \quad . \quad . \quad . \quad . \quad (28)$$

Moreover, from (27) and (28),

$$1-\beta = ka T_1^4/g_1 \quad . \quad . \quad . \quad . \quad (29)$$

7. *Results for a Giant Star.*—Substituting in (27) the values calculated in § 4, we have

$$1-\beta = [5{\cdot}2980]\beta^4.$$

I have given the logarithm of the coefficient. Solving this equation

$$\beta = 0{\cdot}0468,$$

so that less than one-twentieth of the weight is left uncompensated. It will be seen from (20) that $(T_0^4/p_0)_c$ does not depend on the density; thus β is the same for all giant stars of the same mass.

Using this value of β in (29) we obtain

$$k = 29{\cdot}5.$$

This value should presumably be the same for all stars. It indicates that the radiation would be reduced in the ratio $1/e$ after passing through a column containing $\frac{1}{30}$ gm. per sq. cm. section (about 25 cm. of air). This seems possible, since it must be remembered that at the high temperatures concerned the bulk of the radiation is of very short wave-length.

The new values of the central temperature and pressure are

$$T_0 = 7{\cdot}12 \times 10^6 \text{ degrees}$$
$$p_0 = 1{\cdot}19 \times 10^{12} \text{ dynes cm.}^{-2}$$

Surface temperature gradient $= 0°{\cdot}30$ per kilometre.

The total radiant energy imprisoned is

$$H = 2{\cdot}81 \times 10^{47} \text{ ergs}$$
$$= 0{\cdot}238\,\Omega,$$

so that it is no longer an excessive amount.

8. *The Relation between Density and Effective Temperature.*—It appears that a gaseous star must settle itself into a state in which radiation-pressure almost balances gravitation. If the outward flow of radiation is too strong, so that gravitation is overbalanced, the star will blow out, the temperature will fall, and radiation will be reduced. On the other hand, if radiation-pressure falls much below gravitation the star will contract and the outflow of radiation will increase. We can make use of the approximate relation radiation-pressure = gravitation (or the more rigorous equation (29)) to determine the variation of effective temperature with mean density and mass.

Since g_1 (gravity at the surface) is proportional to $M^{\frac{1}{3}}\rho^{\frac{2}{3}}$, we have from (29)

$$T_1^4 \propto M^{\frac{1}{3}}\rho^{\frac{2}{3}}(1-\beta),$$

whence, approximately

$$T_1 \propto M^{\frac{1}{12}}\rho^{\frac{1}{6}} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

The relation is rigorous as regards ρ; and as regards M the correction due to the dependence of β on M is usually insignificant. For example, if M is increased eightfold, formula (30) gives an increase of T_1 in the ratio 1·189, whereas the exact ratio is 1·198.

The range of mass in stars is believed to be small, and will affect T_1 only slightly. The following table shows how T_1 varies with the density for a star of mass 1·5 × Sun.

Effective Temperature.	Mean Density (Water = 1).
13 000°	0·128
10 000	·026
8 000	·007
6 000	·0012
4 500	·00022
3 000	·00002

This seems to correspond fairly well with what we know of the densities and temperatures of the giant stars of different types.*

At 10 000° the central density should be 54·25 × ·026 = 1·4, so that deviations from Boyle's Law will occur near the centre. We may, however, anticipate the results of § 10, and state that it is not until temperatures above 13 000° are reached that the deviations from the gas-law begin to influence the effective temperature.

The total radiation varies as $r^2T_1^4$, which by (29) varies as $g_1r^2(1-\beta)$, that is, as $M(1-\beta)$, and is independent of the density. We thus obtain the result that

The bolometric magnitude of a gaseous star is independent of its stage of evolution, and depends only on its mass.

This is in accordance with H. N. Russell's conclusion that the absolute magnitudes of giant stars of different types are nearly the same. Differences of one or two magnitudes may occur, owing partly to differences of mass and partly to differences of the ratio of the luminous to the total radiation.

From the table the density of a giant star having the same effective temperature as the Sun (6000°) is ·0012. The Sun's density is 1·38. Hence by comparing their superficies I find that a giant star is $5^m·4$ brighter than the Sun, which agrees well with observation. (According to Russell, the average magnitude of a giant star at 10 parsecs distance is probably about zero; that of the Sun is usually given as $5^m·5$, but some recent determinations make it about $0^m·5$ brighter.)

* Russell has given the following estimates of the average densities of giant stars of different types in terms of the Sun's density (*Nature*, vol. xciii. pp. 282–3):—Type A, 1/10; G, 1/350; K, 1/2800; M, 1/25000. Assuming that type G has the same effective temperature as the Sun (6000°), the law $T_1 \propto \sqrt[6]{\rho}$ gives the following effective temperatures:—Type A, 10800°; G, 6000°; K, 4250°; M, 2950°. This agrees almost exactly with the temperatures usually assigned.

Comparison with the Lane-Ritter Theory.—According to Ritter * the effective temperature of a star varies as the square-root of the mass. Our result is the twelfth root, which seems to accord better with the comparatively small range of surface temperatures observed in stars of similar density. Ritter further states that the temperature is nearly independent of the density, whereas according to the present view density is the main determining factor.

Had Ritter taken $\gamma=\frac{4}{3}$, his formula would have led to $T_1 \propto \rho^{\frac{1}{3}}$, agreeing in this respect with (30).

Lane's law, strictly speaking, is not concerned with the effective temperature, but with the temperature at any definite point in the interior; this is proportional to the cube root of the density on Lane's theory as on the present theory. It has often been recognised that the effective temperature will follow a different law; but I do not think the sixth root has been suggested before.

Range of Stellar Masses.—We have found that the total radiation of a giant star varies as $M(1-\beta)$; or, since β is small and varies only slightly, the radiation is practically proportional to the mass. On this view we must attribute the differences in luminosity of individual giant stars of the same spectral type to differences in the masses; and if we knew the range of variation of luminosity we should be able to deduce the range of variation of the masses.

H. N. Russell has given provisionally the necessary data for giant stars of type M.† He has found that these are distributed about a mean absolute magnitude approximately according to the law of errors, the "probable error" being $\pm 0^{m}\cdot 6$. By the law of errors more than four-fifths of the stars will be included within a total range of $2^{m}\cdot 4$ (*i.e.* twice the probable error on either side). The corresponding range of luminosity is 1 : 9; and the range of the masses must be the same.

We conclude that the masses of the stars are dispersed about a median mass M, so that four-fifths of the whole number are between the limits $\frac{1}{3}$M and 3M. This result applies primarily to the giant M stars; but Russell's investigation indicates that the range is very similar in the other types. This remarkably small range of mass is in general agreement with our knowledge derived from observation of binary stars.

9. *Energy of a Star.*—Former investigators have found a difficulty that the heat energy of a star amounts to a large proportion of the energy acquired by contraction, leaving only a small fraction to account for the radiation during past history. Thus Perry showed that if $\gamma<\frac{4}{3}$ the heat (molecular) energy alone is greater than could be acquired by contraction; he neglected the imprisoned radiant energy, which would have made the deficit very much worse. On the present theory this difficulty is avoided, and only about a quarter of the energy of contraction is retained within the star.

* *Astrophysical Journal*, vol. viii. p. 307.

† *Observatory*, vol. xxxvii. p. 170; *Nature*, vol. xciii. p. 255.

Let c_v be the specific heat at constant volume,
γ the ratio of specific heats,
$d\mathrm{V}$ an element of volume,
K the molecular energy of the star,
H the imprisoned radiant energy,
Ω the energy acquired by contraction from infinite diffusion.

Then $$\mathrm{K} = \int c_v \rho \mathrm{T} d\mathrm{V} = \frac{c_v}{\mathrm{R}} \int p d\mathrm{V} = \frac{1}{\gamma - 1} \int p d\mathrm{V},$$

since $$c_v + \mathrm{R} = \gamma c_v.$$

Also $$\mathrm{H} = \int a\mathrm{T}^4 d\mathrm{V} = a\frac{\mathrm{T}_0^4}{p_0} \int p d\mathrm{V} = \frac{3(1-\beta)}{4\beta} \int p d\mathrm{V} \quad \text{by } (26a).$$

Now if there were no radiation-pressure we should have

$$\int p d\mathrm{V} = 4\pi \int p\xi^2 d\xi = -\tfrac{4}{3}\pi \int \xi^3 dp = \tfrac{1}{3}\Omega. \quad (\textit{Gaskugeln}, \text{ p. } 124).$$

But radiation-pressure multiplies the values of p in the ratio β, so that

$$\int p d\mathrm{V} = \tfrac{1}{3}\beta\Omega.$$

Thus $$\mathrm{K} = \frac{\beta}{3(\gamma - 1)}\Omega \qquad \mathrm{H} = \frac{1-\beta}{4}\Omega.$$

We cannot estimate K accurately, since we do not know γ. The material is probably monatomic at the high temperature, but it seems unlikely that the usual value $\gamma = \frac{5}{3}$ is valid for radiating atoms. The factor β in the numerator will make this energy small in almost any case.

Taking Perry's case, $\gamma = \frac{4}{3}$, we have (for mass $= 1{\cdot}5 \times \odot$)

$$\mathrm{K} = {\cdot}047\Omega \qquad \mathrm{H} = {\cdot}238\Omega.$$

Altogether 0·285 of the energy of contraction is retained within the star as heat, leaving plenty of margin for radiation or for the formation of radioactive elements.

It is interesting to note that the greater part (H) of the energy resides in the æther. Only a small part of the star's store of heat is represented by the molecular movements; the bulk of it consists of æther-waves travelling in all directions but unable to escape, except very slowly, through the meshes of matter which imprison them. This condition applies to the gaseous stars only, and not to dwarf stars such as the Sun.

As the star contracts Ω varies as $\rho^{\frac{1}{3}}$, or as T_1^2. Hence, since the total rate of radiation remains constant, we ought to have—

The age of a gaseous star increases as the square of its effective temperature.

I do not place much reliance on this, because it is well known that a star cannot have a reasonably long life if its supply of energy is due solely to contraction.

10. *Dwarf Stars.*—If we were to extrapolate the law $T_1 \propto \rho^{\frac{1}{3}}$ for a star having the density of the Sun, 1·38, we should find an effective temperature of 19 300,° whereas the Sun's effective temperature is about 6000°. The difference is due to the failure of the gas-law at this high density, which renders the calculated values of $(T_0^4/p_0)_c$ erroneous. It should be noted that we have not used the gas-law in deriving equations (27), (28), (29), or indeed in any of the numbered equations except (20). Thus the whole theory holds for the dwarf stars, except that the calculations of § 4 break down.

The effect of a failure of the gas-law is to alter the value of $(T_0^4/p_0)_c$, which by (27) alters β. β begins to increase. By (29) T_1 varies as $\sqrt[4]{(1-\beta)}$. Since β is at first about 0·05, the increase of β two- or three-fold makes at first but little difference in this factor. It is not until long after the internal distribution of temperature and density has deviated strongly from that given by the gas-law that T_1 begins to be affected.

It is of great interest to determine the turning-point at which the effective temperature ceases to increase with the density.

For high densities and pressures the necessary modification of Boyle's Law is given approximately by Van der Waal's equation, and the gas-equation becomes

$$p = R\rho T\left(1 - \frac{\rho}{\rho_0}\right)^{-1} \quad . \quad . \quad . \quad . \quad (31)$$

where ρ_0 is the maximum possible density of the material. This equation must be combined with (26*a*)

$$p = \tfrac{4}{3}a\frac{\beta}{1-\beta}T^4 \quad . \quad . \quad . \quad . \quad (32)$$

and

$$\frac{1}{\rho}\frac{dp}{d\xi} = -\beta g \quad . \quad . \quad . \quad . \quad . \quad (33)$$

to determine the interior conditions. An exact solution is, of course, impossible; but we can without much difficulty attain our object by quadratures.

Since it is clear that on this theory the Sun must be compressed to something near the maximum density throughout most of its mass, I take ρ_0 to be 1·5.* Our object is—given the mass and mean density, to find β. We have to proceed by guessing β and then working from the outside inwards with equations (31), (32), and (33) to test whether the value fits.

For a star of the same mass as before, and density 0·7, I find that β must be 0·18. The following table gives the radii of the surfaces of equal density, the mass contained within them, the temperature, and the acceleration of gravity there.

* N. Ekholm gives 1·486. His neglect of radiation-pressure and other divergences will make no serious difference in the case of the Sun. See *Gaskugeln*, p. 467.

Dense Star.

(Mass = 1·5 ⊙ ; Density = 0·7.)

Density.	Radius.	Interior Mass.	Temperature.	Gravity.
0	$9{\cdot}92 \times 10^{10}$	$2{\cdot}87 \times 10^{33}$	...	$1{\cdot}94 \times 10^{4}$
0·05	9·32	2·87	$3{\cdot}4 \times 10^{6}$	2·20
0·15	9·05	2·85	5·0	2·31
0·25	8·87	2·81	6·1	2·38
0·35	8·70	2·76	7·0	2·43
0·45	8·54	2·70	7·8	2·46
0·55	8·37	2·62	8·6	2·49
0·65	8·18	2·52	9·5	2·51
0·75	7·95	2·39	10·4	2·52
0·85	7·67	2·22	11·3	2·51
0·95	7·29	1·98	12·5	2·48
1·05	6·76	1·65	13·8	2·41
1·15	5·86	1·16	15·4	2·25

The mass remaining within the last sphere must correspond to a mean density greater than 1·15, and less than the limiting density 1·5. The actual value deduced from the table is 1·37, and this constitutes our test that the value of β is correct—we have stripped off just the right amount. Had we chosen $\beta = 0{\cdot}20$, the interior density would have been less than 1·15 ; similarly a smaller β would have given a density greater than 1·5.

According to formula (30) this star would have an effective temperature of 17 300° ; the factor $(1-\beta)^{\frac{1}{4}}$ reduces this to 16 600°. It is remarkable that such an extreme deviation from the conditions of a perfect gas should lead to so small a correction. It is clear that 16 600° must be about the turning-point of the temperature, because when once the term in β has begun to take effect it will soon counterbalance any increase of density.

By (30) the maximum effective temperature will be proportional approximately to the twelfth root of the mass. The correction depending on β can be determined from (27), since *for the same mean density* $(T_0^4/p_0)_c$ varies as M^2, whether the star be gaseous or otherwise (*cf.* (20)). Thus for the same density

$$\frac{1-\beta}{\beta^4} \propto M^2.$$

Introducing this correction, I find the following results :—

Mass = 4·5 × Sun,		Maximum temperature =	18 600°
,,	1·5 ,,	,, ,,	16 600°
,,	0·5 ,,	,, ,,	14 600°

These values are perhaps too high. The observational evidence of Nordmann and of Wilsing and Scheiner seems to point to a maximum of about 14 000°. But the agreement seems sufficiently

close to be of interest. Lower maxima, in the ratio five-sixths, would have been obtained if Russell's densities (*footnote*, p. 29) had been used instead of the datum density 0·002 corresponding to temperature 6500°.

The dwarf-stars must be comprised within the range of density 0·7 to 1·5. The importance of radiation-pressure compared with gravitation decreases rapidly as we descend along the dwarf series. For the Sun, radiation-pressure is practically negligible. The differences of luminosity of the dwarf types must be due mainly to differences of effective temperature, since there is not much scope for variation of superficial area.

11. *Molecular Weight.*—The calculations have been based throughout on a molecular weight 54. To illustrate the effect of modifying this, I give some results for a molecular weight 18. The gas-constant R becomes multiplied by 3. The central pressure becomes multiplied by 3 very approximately, the central temperature being almost unaltered. The new value of β is ·137, so that about $\frac{6}{7}$ of gravity is compensated by radiation-pressure, instead of $\frac{19}{20}$. The constant of absorption k is slightly reduced to 26·7. The results of § 8 are based on the general relation $T_1 \propto M^{\frac{1}{3}}\rho^{\frac{1}{3}}$, and are not altered. The calculations of the internal energy of the star in § 9 involve β, and the numerical results are altered, but they are of the same general character; the molecular energy is, however, increased and the radiant energy slightly diminished.

The most important effect of diminishing the molecular weight to 18 is to reduce the maximum possible effective temperature and to cause it to occur at a lower mean density. It is not possible to give accurate figures without a great deal of labour, but it appears that the maximum effective temperature would be decreased by about 1200°.

The importance of radiation-pressure in determining stellar equilibrium would be reduced if the molecular weight were still smaller; but even for a star composed of helium, it would still be intense enough to counterbalance half the gravitation.

12. *The Constant of Absorption.*—We have found that k has a value about 30 in C.G.S. units, and this should be the constant of absorption for stellar material at temperatures chiefly between 10^6 and 10^7 degrees prevailing within stars. Unfortunately, there seems no other evidence to indicate whether this value is at all near the truth. At temperatures of 10^6 and 10^7 degrees the radiation of maximum intensity is of wave-length 30 and 3 tenth-metres respectively—shorter than ordinary spectral radiation, but longer than X-rays and γ-rays. It has one merit for our purpose—it cannot very well experience selective absorption, since no spectrum (X-ray or ordinary) contains lines within this range.

It happens that the experimental values of k for hard X-rays absorbed by solid material are usually about equal to the value here found.* In the absence of a theory to guide us, we cannot

* Bragg, *X-Rays and Crystal Structure*, p. 177. Table A gives k, defined as in this paper.

infer that the same values would hold at a temperature of 10^6 degrees and for considerably longer waves. The agreement is probably accidental. The work which seems to bear most nearly on the question is J. J. Thomson's theory of the scattering of X-rays.* For very short waves the scattering depends only on the number of electrons present—for a gas, practically on the mass. The theory, confirmed by the experiments of Barkla, gives a scattering of ·00025 by a cubic centimetre of air. This is equivalent to $k = 0{\cdot}2$ only; but this is only the part of the coefficient due to scattering, and the full absorption coefficient must be larger.

Experimental results for the extreme ultra-violet have, I think, no application to our problem, for the absorption may be molecular and is not likely to persist at high temperatures.

The Stellar Magnitude Scales of the Astrographic Catalogue. Tenth Note. The Melbourne Magnitudes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. During the visit of the British Association to Australia in 1914 we were glad to learn that the measurement and reduction of the Melbourne zones ($-65°$ to $-90°$, the portion corresponding to that undertaken by Greenwich in the northern hemisphere) were nearing completion, and that funds had been provided for the printing. It is earnestly to be hoped that neither the war nor the retirement of Mr. Baracchi from the Directorship will affect the availability of these funds, and that the printing may go forward rapidly. The Council of the British Association, after consultation with the Council of the Royal Astronomical Society, drew the attention of the Victorian Government to the importance of this printing, expressing great satisfaction that the means had been promised.

2. Meanwhile Mr. Baracchi, in kind response to a request, sent me the counts of the plates in zone $-65°$; material of great value, seeing that nothing similar has yet been published south of $-42°$ (Cape). Experience of the Vatican plates in similar latitudes and of the Oxford plates indicates that the results of a single zone are liable to be sensibly affected by accidental errors (*Mon. Not.*, lxxv. pp. 468 and 603). Hence the following results may possibly be modified when other zones are available.

3. We will depart a little from previous procedure by considering first the magnitude scale as a whole. In Table I. are given, first, d, the diameter as measured at Melbourne in units of $0''{\cdot}25$; next, N′, the total number of stars on all the plates in the zone with a diameter greater than d; and next, log N, the average *per plate*, *i.e.* log N′ − log 80, there being 80 plates in the zone.

* Richardson, *Electron Theory of Matter*, p. 485.

Using then the scale of Chapman and Melotte (as expanded in *Mon. Not.*, lxxv. p. 145), we get the magnitudes under the heading C. & M. Then follows the $\sqrt{d}$: and on adding this column to

TABLE I.

General Magnitude Scale for Zone – 65° (Melbourne).

d>.	N′.	log N.	C. & M.	$\sqrt{d}$.	Sum.	Calc. d.	O – C.
55	84	0·02	7·10	7·40	14·50	54·3	+0·7
50	117	0·17	7·40	7·05	14·45	50·0	0·0
45	189	0·38	7·80	6·70	14·50	44·5	+0·5
40	265	0·52	8·10	6·35	14·45	40·6	−0·6
35	470	0·77	8·60	5·90	14·50	34·5	+0·5
30	735	0·97	9·00	5·50	14·50	29·9	+0·1
25	1169	1·17	9·45	5·00	14·45	25·2	−0·2
20	1864	1·37	9·90	4·50	14·40	20·9	−0·9
15	4863	1·79	10·90	3·85	14·75	12·7	+2·3
10	15,675	2·30	12·25	3·15	15·40	4·9	+5·1
All	33,813	2·63	13·20	...	...	1·6	...

the former we see that the results are nearly constant until we come to the faintest stars, so that magnitudes may be obtained from the Melbourne magnitudes by the formula

$$m = 14{\cdot}47 - 1{\cdot}00\sqrt{d}$$

with considerable accuracy down to $d = 20$. For diameters smaller than this, however, the law ceases to be applicable. The numbers in the last column but one show the actual diameter corresponding to the value of log N on the scale of C. & M.: and the differences O – C between this and the first column are satisfactorily small.

4. But now these diameters were measured on three different machines, two of Repsold's and one "slide" machine. Mr. Baracchi sends the following note:—

"On the slide machine all diameters over 16 were measured, on Repsold I. all diameters over 24, and on Repsold II. all diameters over 50. Repsold I. has two parallel wires and Repsold II. four parallel wires in each co-ordinate which served as standards for the estimation of diameter. The unit used in the table is $\frac{1}{4}$ second of arc, the original estimates being made to quarter seconds. In the tables the results for the northern and southern halves of each plate are given separately. The measured area is a square whose side is 26 réseau intervals, the value of one réseau interval being $5' \times (1 - 1/87)$."

5. It is therefore important to see whether the diameters found by the three machines accord. The plates measured by the three machines are fairly well scattered through the 24^h, and in the first instance we may adopt the plan of comparing simply the totals for

each instrument, or rather the mean totals per plate, and it will suffice to give the log totals, which occupy the 2nd, 3rd, and 4th columns of Table II.

TABLE II.

Log N for the Three Different Machines R_1, R_2, S.

$d>$.	R_1.	R_2.	S.	R_2-R_1.	$\frac{1}{2}(R_1+R_2)$.	$S-\frac{1}{2}(R_2+R_1)$.	$-0\cdot44$.
55	9·93	9·76	0·32	−0·17	9·84	+0·48	+0·04
50	9·97	0·01	0·47	+0·04	9·99	+0·48	+0·04
45	0·19	0·24	0·65	+0·05	0·22	+0·43	−0·01
40	0·35	0·32	0·83	−0·03	0·34	+0·49	+0·05
35	0·58	0·67	1·03	+0·09	0·62	+0·41	−0·03
30	0·80	0·86	1·22	+0·06	0·83	+0·39	−0·05
25	1·04	1·02	1·43	−0·02	1·03	+0·40	−0·04
20	1·24	1·28	1·59	+0·04	1·26	+0·33	−0·11
15	1·77	1·76	1·84	−0·01	1·76	+0·08	−0·36
10	2·31	2·18	2·41	−0·13	2·25	+0·16	−0·28
All	2·64	2·59	2·66	−0·05	2·62	+0·04	−0·40

6. In the 5th column are shown the differences $R_2 - R_1$. There is a trace of systematic error near the bottom: but the accordance is good on the whole, and we may fairly take the mean of the two results, as in the 6th column, to represent the Repsold machines.

7. But when we compare this with S as in the 7th column there are striking differences, which average 0·44 at the top and fall to nearly zero at the bottom. Now it is a fair presumption that the average number of stars per plate is the same for the plates measured by each machine, in which case the small differences at the bottom are presumably correct. But what are we then to make of the very considerable differences at the top? They are the more remarkable because, according to Mr. Baracchi's note, the diameters of the brighter stars were deliberately *measured*, while those of the fainter were estimated only. Hence the first possibility considered was that, after all, the bottom differences must be wrong, and that we should subtract 0·44 from all the numbers in the 7th column (as shown in the 8th column), representing the difference between the average number of stars on plates measured in the S machine and those measured in the R machines.

8. Since log 3 = 0·48, it follows that the S plates contain nearly three times as many stars as the R plates. But this is so large a discrepancy that it should be easily discernible. To test its reality the number of stars on every S plate was compared with the mean of the R plates immediately preceding and following it. The totals were—

S = 8509	R = 7041	
log = 3·93	log = 3·84	Diff. = +0·09.

The difference 0·09 accords fairly well with the run of the numbers near the bottom of column 7, Table II., and is quite inconsistent with any such systematic difference as 0·44. We are thus led to consider these differences for bright stars as real, and their remarkable character is shown by the consideration that they involve calling a diameter $d = 42$ on one machine and $d = 55$ on another.

9. It was natural to write to Melbourne to see what light could be thrown on the discrepancy, and at my request Mr. Baldwin very kindly had three plates which had been measured on the S machine remeasured (as regards diameters only) on R_2. But it may fairly be said that these remeasures only increase the mystery. There is some evidence of personality between observer P, who made all the remeasures on machine R_2, and observer S, who measured one plate on machine S. But it does not occur in the right place in the scale, and the personality between P and the four other measurers (T, MH, L, and P) who first measured the other two plates is insensible.

The following are the values of log N:—

TABLE III.

	Plates $1^h\ 39^m$ and $5^h\ 33^m$.			Plate $14^h\ 33^m$.		
$d >$.	S.	R_2.	S − R_2.	S.	R_2.	S − R_2.
50	0·84	0·70	+0·14	0·30	0·48	−0·18
45	1·00	1·00	·00	0·84	0·84	·00
40	1·18	1·11	+ ·07	1·04	0·95	+ ·09
35	1·34	1·36	− ·02	1·18	1·15	+ ·03
30	1·38	1·40	− ·02	1·42	1·23	+ ·19
25	1·52	1·45	+ ·07	1·73	1·49	+ ·24
20	1·66	1·59	+ ·07	2·00	1·63	+ ·37
15	2·03	1·96	+ ·07	2·27	1·89	+ ·38
10	2·53	2·48	+ ·05	2·58	2·53	+ ·05
All	2·74	2·74	·00	2·71	2·71	·00

The numbers of bright stars concerned are of course small, and this evidence may not be conclusive. But the systematic difference is so large that the origin of it ought to be traceable with little difficulty.*

10. Although the existence of this systematic error may somewhat diminish the value of our conclusions, we will now see what this single Melbourne zone (the only part of the *Astrographic Catalogue* yet to hand, even in MS., from − 52° to the South Pole) tells us about the stellar distribution in R.A.

11. In Table IV. are given the values of the counts for each

* Mr. Baldwin's letter is dated 1916 January 13, and I have delayed this note a little in the hope that further information may be forthcoming: but nothing more is yet to hand.

hour of R.A. at certain definite limits of diameter. Under $d > 40$ are given the total numbers of images with diameters of 41 or more: and so for the other columns. The ninth column, headed "All," gives the total number of stars on the plates in each hour. Two-thirds of the "hours" contain three plates each, while hours 1^h, 4^h, 7^h, etc., contain four plates: but as in what follows we are chiefly concerned with log differences for each hour, from which a factor 3 or 4 disappears, the simple counts have been left as they stand, to avoid the use of fractions.

TABLE IV.

Cumulative Totals (N') for Melbourne − 65° for each Hour of R.A.

$d >$.	40.	35.	30.	25.	20.	15.	10.	All.	(α)	(β)
0^h	2	7	9	18	32	77	265	361	− 9	+ 1
1	9	19	26	29	44	107	345	554	− 4	−29
2	7	9	12	16	28	73	242	314	− 9	−16
3	3	5	8	18	31	94	299	391	−11	+10
4	2	9	21	36	73	182	545	745	− 9	− 4
5	12	16	19	36	51	146	448	685	− 6	− 6
6	6	17	25	40	61	189	615	1058	− 5	− 3
7	10	21	32	43	63	223	813	1796	+ 1	+ 4
8	6	14	23	37	74	200	430	1251	+12	+ 2
9	23	35	54	84	131	266	855	1779	+ 2	−15
10	41	66	105	139	219	457	1568	3532	+ 3	−16
11	3	4	10	24	37	103	386	1494	+16	+38
12	31	57	99	169	238	420	1448	3040	+ 2	−19
13	15	25	35	70	119	422	1619	3356	− 2	+28
14	21	33	51	88	168	363	990	1443	− 6	−14
15	9	18	29	42	80	357	1140	3074	+ 6	+29
16	27	48	66	103	152	403	1059	3035	+10	− 6
17	5	10	13	16	26	114	472	1029	0	+19
18	7	11	20	37	56	203	607	1765	+ 9	+20
19	9	17	29	45	56	126	449	841	− 1	−19
20	4	7	12	20	30	83	290	654	+ 2	+ 3
21	5	8	13	24	35	92	302	668	+ 2	+ 2
22	5	8	15	23	40	115	354	704	0	0
23	3	6	9	12	20	48	134	244	− 1	−20
Totals	265	470	735	1169	1864	4863	15675	33813		
Mag. C.&M.	8·10	8·60	9·00	9·45	9·90	10·90	12·25	13·20		

12. In the last two columns are given—

under heading (α), log "All" − log "10" − 0·56,
and under heading (β), log "All" − log "30" − 1·93,

log "10" denoting log N′ for the hourly total in the column $d > 10$, and the constants 0·56, 1·93 being the mean values of the log differences for the 24 hours. From the magnitude scale given at the foot of the table we see that the column (α) compares mags. 13·20 and 12·25, differing by 0·95 mag.: and column (β) compares 13·20 and 9·00, a difference of 4·20 mags.

13. For harmonic analysis of these columns it will suffice to add together 23^h and 0^h, giving a total applicable to the midpoint $0^h{\cdot}0$: with similar totals for the points $2^h{\cdot}0$, $4^h{\cdot}0$, etc., a set of 12 terms, which is quickly dealt with. The first harmonics are (applicable to hourly results)—

$$\text{for } (\alpha),\ -3{\cdot}5 \sin\theta - 4{\cdot}9 \cos\theta = -\ 6{\cdot}0 \cos(\theta - 36^\circ),$$
$$\text{for } (\beta),\ -7{\cdot}5 \sin\theta - 6{\cdot}7 \cos\theta = -10{\cdot}0 \cos(\theta - 49^\circ).$$

We may perhaps add an intermediate determination, comparing mags. 13·20 and 9·90 ($d > 20$), giving

$$-8{\cdot}7 \sin\theta - 4{\cdot}8 \cos\theta = -10{\cdot}0 \cos(\theta - 61^\circ).$$

14. Now the phases 36°, 61°, and 49° are fairly accordant: the mean value is 48°, meaning that the "obscuration" is greatest at about $3^h{\cdot}0$. To represent the effect as an obscuration of faint stars the coefficients −6·0, −10·0, and −10·0 have been given with negative signs (as proposed on p. 154 of *M.N.*, lxxvi.). But the coefficients do not increase as fast as the difference of magnitude, there being no change at all between mags. 9·00 and 9·90, a change of 4·0 between mags. 9·90 and 12·25, and a change of 6·0 in the short interval 12·25 to 13·20. This, however, is in good accordance with the Cape counts, which do not show any sensible "obscuration" effect until we reach faint stars (see Table XVI. on p. 480 of *M.N.*, lxxv.).

15. We may now see how these results fit the "spiral" suggested (*M.N.*, lxxvi. p. 154 or 157).

We find

$$\begin{aligned}\psi_0 &= A + 3{\cdot}66\delta - 247^\circ,\\ &= 48^\circ - 3{\cdot}66 \times 65^\circ - 247^\circ,\\ &= -77^\circ.\end{aligned}$$

The Cape zones −49°, −50°, and −51° give $\psi_0 = +39^\circ$, -15° and -93°, so that there is some hint of a negative value. But, in the absence of intermediate zones, we cannot yet say anything definite. It is much to be regretted that we have as yet so little material in this part of the sky: a single zone from the Sydney Observatory would be of great value. To supply the deficiency counts have been made of the *visual* magnitudes observed at Cordoba, and will be given in another note. But an appeal is hereby made to the Directors of the Melbourne and Sydney observatories for more material as soon as it can be furnished, either in print or in MS., but preferably, of course, in print.

16. Meanwhile another point may be noticed about the Melbourne results which seems to strengthen the evidence for the

"spiral." It was shown on p. 153 of *M.N.*, lxxvi., that when the circuits of R.A. are started with ψ_0 in each case, one edge of the spiral is much sharper than the other, the peak occurring at 15^h from ψ_0. Now the same character attaches to the Melbourne results. There are, of course, large accidental irregularities which mask it to some extent; but they can be partly nullified by grouping. Let us, for instance, take two hours of R.A. together, and also combine columns (α) and (β) of Table IV. We will start with the group $3^h + 4^h$, the mean of the four numbers $-11 - 9 + 10 - 4$ in columns (α) and (β) being -4; and we then follow round with similar groups of 4, as below:—

	(0h).	(2h).	(4h).	(6h).	(8h).	(10h).	(12h).	(14h).	(16h).	(18h).	(20h).	(22h).
M	−4	−5	+5	−6	+9	+2	+10	+12	−4	+1	−7	−14
C	−4	−3	−3	0	+4	+8	+11	+16	−1	−9	−10	−10

The Melbourne numbers are given in the row M: in the row C are given the simple sums, for every two hours, of the last column of Table IV. on p. 153 of *M.N.*, lxxvi.

It will be seen that the numbers run together very fairly. If we again add every consecutive pair to get six groups, we find—

M	−9	−1	+11	+22	−3	−21
C	−7	−3	+12	+26	−10	−20

17. The unit of magnitude for Melbourne has been arbitrarily chosen to fit, so that the similarity of the scales has no significance: it is the *arrangement* of the numbers in circuit to which attention is called. We must, however, justify the starting-point chosen for Melbourne: the Cape zones have been started with the epoch of greatest obscuration, which for Melbourne we have found to be 48° or close to $3^h{\cdot}0$: hence to correspond as nearly as possible with the Cape arrangement we must begin with 3^h, as in fact we did.

18. Finally, one word more may be said as to the systematic difference between the machines. We might perhaps have tried the effect of correcting S to accord with R_1 and R_2, but Table II. shows that if the distribution of S measures round the circuit had any sensible effect on the phenomenon under consideration, it would be shown more markedly in the case of $d > 30$ and $d > 20$ than in that of $d > 10$. Since all three phases are in good accord, we may fairly conclude that to make this correction would be an unnecessary refinement at the present stage.

Summary.

(α) Counts of the Melbourne zone -65° show that the recorded diameters d of the star images can be converted into magnitude (C. and M. scale) by the formula

$$\text{mag.} = 14{\cdot}47 - 1{\cdot}00\sqrt{d},$$

the constant 14·47 being a mean value only, but the coefficient of $\sqrt{d}$ being probably applicable to the plates generally (§§ 1–3).

(β) There is, however, a systematic difference between measures made on the S machine and those made on the two R machines, which calls for further investigation (§§ 4–9).

(γ) Analysis of the R.A. circuit shows that the "greatest obscuration" is at $3^h{\cdot}2$, giving

$$\psi_0 = 48^\circ - 3{\cdot}66 \times 65^\circ - 247^\circ = -77^\circ.$$

(δ) The "obscuration" only begins among the faint stars —say after mag. 10 or 11,—as was found for the Cape zones (§ 14).

(ϵ) The following edge of the *unobscured* spiral is the sharper, the distribution in R.A. or ψ being similar to that shown by the Cape zones (§§ 16, 17).

An Observation by Lamont of Professor Barnard's Proper Motion Star. By A. C. D. Crommelin, B.A., D.Sc.

The discovery by Professor Barnard of a faint star with a proper motion of more than 10″ per annum is of such great interest that it seems worth while to point out that the star was almost certainly observed by Lamont at Munich on 1842 July 24. (It is No. 15040 in Munich (1) 1880.) The only reason for the smallest doubt in the matter is that there is another faint star within some 14″ of the position recorded by Lamont. The establishment of identity involves (1) the proof from independent sources that Barnard's star was within a few seconds of arc of Lamont's position, (2) the proof that Barnard's star is at least half a magnitude brighter visually than the other star.

Lamont's observations were made in zones of declination about $1\frac{1}{2}^\circ$ in width, a graduated sector being used whose position relatively to the declination circle could be changed from night to night. As his readings on the night in question show that the sector shifted 13″ between the beginning and the end of the zone, it was necessary to re-reduce the observations to find when the shift occurred. The zone consists of 80 stars, the Barnard star being the eighth. The method adopted was to carry back the places of the Albany A.G. Catalogue to the epoch 1842 with the aid of proper motions supplied to me by Mr. Thackeray, which were deduced from comparison of Albany with Abbadia. I have used the seven stars preceding the Barnard star and seven following. They indicate corrections to Lamont as follows:—

Star.	P.D. Mag.	Corr. to Lamont. R.A.	Decl.	Star.	B.D. Mag.	Corr. to Lamont. R.A.	Decl.
		s	″			s	″
B.D. +3°3493	6·9	−·04	−0·7	B.D. +3°3552	9·2	−·38	−2·7
+4·3517	7·7	−·39	+1·3	3·3562	8·7	−·30	+1·4
+4·3530	9·0	−·30	+1·2	4·3573	8·5	+·46	−0·6
+4·3541	7·0	−·19	+1·2	4·3574	9·2	−·46	+2·8
+4·3542	8·6	+·34	+0·7	4·3602	9·0	−·22	−0·5
+4·3545	8·7	+·17	+2·7	3·3597	8·7	−·24	−1·7
+3·3533	8·1	−·42	−0·7	3·3611	7·9	−·19	−1·0

The mean correction is $-^{s}\cdot15$, $+0''\cdot2$; the average discordances are $0^{s}\cdot20$, $1''\cdot4$, which are not unexpectedly large considering that transits were observed over only one wire. It is clear that the shift in the sector did not occur till after these stars had been observed. This is confirmed by the fact that Lamont himself bracketed the declinations of all the stars at the end of the zone; apparently it occurred between the 67th and 68th star.

Lamont's place of the Barnard star for 1842·0 is

$$17^{h}\ 50^{m}\ 7^{s}\cdot09 + 4^{\circ}\ 16'\ 4''\cdot7.$$

Applying the corrections $-^{s}\cdot15$, $+0''\cdot2$ and reducing to the equinox of 1916·0, we obtain

$$17^{h}\ 53^{m}\ 46^{s}\cdot72 + 4^{\circ}\ 15'\ 12''\cdot8, \text{ epoch } 1842\cdot56.$$

Comparing this with the position from Greenwich meridian observations, reduced without proper motion to the equinox of 1916·0, viz. $17^{h}\ 53^{m}\ 43^{s}\cdot49 + 4^{\circ}\ 27'\ 49''\cdot8$, epoch 1916·59, we obtain the proper motion $-^{s}\cdot0436, +10''\cdot225$, or in great circle $10''\cdot25$ in P.A. 356°·35. Professor Barnard's value as printed in *Astr. Journ.*, 695, was $10''\cdot30$ in P.A. 359°·7, but the latter angle appears to be affected by a clerical error, and should read 356°·7. This will be seen if the P.M. be re-determined from Professor Barnard's data. As the error is repeated in *Popular Astronomy* for October, it is as well to call attention to it.

As a further check I measured, with the aid of a microscope, the Toulouse Astrographic Chart of 1898 July 25. (The catalogue of this region is not yet to hand.) The distance of the Barnard star from the star that Professor Barnard denotes by *n* was found to be $15''\cdot9$ in position-angle 104°·5. I assumed for *n* the position $17^{h}\ 53^{m}\ 43^{s}\cdot31 + 4^{\circ}\ 24'\ 48''\cdot4$ (eq. 1916·0), derived from Professor Barnard's micrometer measures, and the Abbadia position of B.D. +4°, 3560. This gives for the Barnard star on 1898 July 25 $17^{h}\ 53^{m}\ 44^{s}\cdot34 + 4^{\circ}\ 24'\ 44''\cdot4$ (eq. 1916·0).

From Lamont, Toulouse, we obtain P.M. $10''\cdot23$ in P.A. 356°·43.
From Toulouse, Greenwich, „ $10''\cdot31$ „ 356°·03.

Since the spectroscopic results of Professors Adams, Campbell, and Slipher indicate a speed of approach of the order of 100 kms.

per second, we may reasonably expect an appreciable secular increase of the P.M. owing to diminished distance.*

Having now proved that the Barnard star was within a few seconds of arc of the object observed by Lamont, it remains to consider the question of magnitude. Lamont assigns the magnitude 10 to the Barnard star, but his magnitudes require a correction to reduce to the B.D. scale. I compared his magnitudes of all stars observed on 1842 July 24 with the B.D. magnitudes, with the following result:—

Mag. given by Lamont .	7–8	8	8–9	9	9–10	10
Mean of B.D. mags. of stars	5·7	...	7·3	7·7	8·2	8·7
Number of stars. . .	1	0	2	11	11	54

There is a reasonable presumption that the object observed by Lamont was not fainter than 9·4 or 9·5 on the B.D. scale. He observed scarcely any stars that are not in the B.D.

Professor Bauschinger revised Lamont's zones at Munich about 1886, using the same instrument (aperture $4\frac{1}{5}$ inches), but without the zone sector. In seeking for the star 15040 he naturally did not find it, as it was over 7′ distant from its former place, but he observed a fainter star in the same region on two nights, 1886 July 18 and August 12. (It is No. 6966 in Munich (2) 1880.) I re-reduced his observations of each night in the same way as those of Lamont, and obtained (eq. of 1880·0) —

	Observed R.A.	No. of Wires observed.	Observed Decl.
	h m s		° ′ ″
1886 July 18	17 51 59·50	1	+4 15 52·1
1886 Aug. 12	17 51 59·96	2	4 15 46·0

Giving the second R.A. double weight, we obtain for the equinox of 1916·0, $17^h\ 53^m\ 46^s{\cdot}83 + 4^\circ\ 15'\ 26''{\cdot}7$. Assuming that this star has no P.M. (I have ascertained from the Toulouse chart and from recent measures that its P.M. is small), in 1842 July it was 14″ north of the Barnard star. The two stars must have been very near each other at the end of 1843.

Professor Bauschinger gave the magnitude of 6966 as 11·0 and 10·0 respectively on the two nights. Comparison of his magnitudes of faint stars with those in the B.D. indicate a correction of $-0^m{\cdot}6$ on each night, so that the mean magnitude of 6966 on the B.D. scale is 9·9, exactly the same as the value independently assigned to it by Mr. W. H. Steavenson last July.

Professor Bauschinger noted on each night that 6966 was too faint to observe accurately, although the observing conditions were described as good. It is therefore quite unlikely that Lamont would have selected such a faint star for observation.

* Taking the parallax 0″·50 given by Schlesinger, the annual P.M. would increase 0″·05 per century.

The Barnard star could be observed comfortably with the Greenwich Transit Circle with illuminated field, so that its magnitude on the B.D. scale can scarcely be fainter than 9·4 or 9·5. Mr. W. H. Stéavenson, who kindly observed the region at my request, made it 9·4, and 6966 just half a magnitude fainter. My own experience of magnitude-estimation is small, but so far as it goes it fully confirms these figures.

The photographic magnitude of the Barnard star on the Toulouse chart is given as 10·5 by Mr. Innes, 10·8 by Mr. Melotte, 11·0 by myself. Being of spectral type Mb, its colour index would be nearly 2 magnitudes, so that its visual magnitude cannot be much fainter than 9. Star 6966 appears about $10^{m}\cdot9$ on the chart, and has probably a smaller colour index than the Barnard star.

The question remains, whether the Barnard star or 6966 is identical with B.D. $+4^{\circ}\cdot3561$. I cannot ascertain exactly when this region was surveyed at Bonn, but it appears to have been either in 1852 or 1853.

In the summer of 1852 the position of the Barnard star (eq. of 1855·0) was

$$17^{h}\ 50^{m}\ 45^{s}\cdot11 + 4^{\circ}\ 17'\ 36''\cdot3,$$

that of star 6966 was

$$17^{h}\ 50^{m}\ 45^{s}\cdot49 + 4^{\circ}\ 16'\ 8''\cdot0,$$

the B.D. place of $+4^{\circ}\cdot3561$ is

$$17^{h}\ 50^{m}\ 42^{s}\cdot9 + 4^{\circ}\ 16'\cdot9.$$

It will be seen that the Barnard star is the nearest to the B.D. place; owing to this, and its greater brightness, it would seem to have the greater claim to be considered identical with $4^{\circ}\cdot3561$. But, considering the low power employed in the Durchmusterung, the position may possibly refer to the mean of the two stars, considered as a single entity.

If one desires an old designation for the Barnard star, I submit that the name Munich (1) 15040 may be adopted with tolerable certainty, or B.D. $+4^{\circ}\cdot3561$ with considerable probability.

55 *Ulundi Road, Blackheath, S.E.:*
1916 *November* 6.

[*Note added* 1916 *Dec.* 9.—A further confirmation of the identity of Lamont's star is afforded by the Proper Motion published by M. Gonnessiat in *Comptes Rendus* for 1916 Nov. 27, viz. $10''\cdot286$, in P.A. $356^{\circ}\cdot07$, deduced from Astrographic plates taken at Algiers 1897 Aug. 4, 1910 June 15, 1916 Oct. 7. Owing to the large scale of the plates, and the smaller star images, this is more trustworthy than any of the other photographic determinations.]

Micrometrical Measures of 250 Double Stars.

By W. S. Franks.

The following measures of Σ stars were made with the Cooke position micrometer on the 6½-inch equatorial at Mr. Frederick J. Hanbury's observatory, Brockhurst, East Grinstead. They form a continuation of the list published in vol. lxxvi., part 1, of the *Monthly Notices*—and the same remarks as there given will also apply. A few objects from OΣ's catalogue, left over from a previous list, are included, their numbers being placed within brackets.

Measures of Double Stars.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
26 *rej.*	0 15·8	+66 27	7·2, 9·5	114·4	13·58	1915·678	2	wh, prpsh.
[OΣ 11 *rej.*]	,, 25·4	31 35	7·8, 7·9	317·4	197·11	·678	2	wh, wh.
46	,, 34·6	20 53	5·0, 8·2	193·1	6·49	·688	2	55 Pisc. ; orge, bl.
47	,, 35·1	23 30	6·7, 8·6	205·0	16·54	·687	2	AB. wh, bl.
60	,, 43·1	57 17	4·0, 7·6	253·3	6·44	·687	3	η Cass. ; p. yel, ruddy purp.
79	,, 54·5	44 13	6·0, 7·0	192·4	7·74	·688	2	wh, grnsh.
127	1 26·8	78 37	8·0, 9·0	186·9	24·20	·715	2	
180	,, 48·0	18 48	4·2, 4·4	359·4	8·04	·728	2	γ Ari. ; v. wh, blsh wh.
188 *rej.*	,, 52·2	62 26	Fr.8·5, 9·5	236·8	33·18	·715	2	p. orge, bl (not in Lewis).
205	,, 57·7	41 51	2·5, 5·5	63·2	10·11	·728	2	γ And. ; gold yel, grnsh bl.
219	2 4·4	32 53	8·2, 9·0	182·6	11·63	1916·014	2	
237	,, 10·4	10 19	8·4, 8·7	238·1	14·35	·029	2	
238	,, 11·2	37 1	8·3, 9·0	357·3	10·92	·029	2	
270	,, 23·8	55 7	7·2, 9·0	302·7	21·18	·029	2	wh, blsh.
281	,, 30·6	5 9	5·0, 9·6	83·1	7·62	·050	2	ν Ceti ; yel, bl.
296	,, 37·3	48 48	4·2, 9·5	302·1	18·00	·050	2	AB. p. yel, two blsh. θ Persei. } Triple.
,,	,, ,,	,, ,,	,, 10·0	227·8	75·44	·050	2	AC. A and D< in both. } Triple.
318	,, 47·6	37 56	5·2, 9·2	237·6	14·02	·020	2	20 Pers. ; wh, bl.
332	,, 52·7	0 1	8·5, 8·5	53·3	12·24	·029	2	
336	,, 55·4	32 1	6·5, 8·0	8·2	8·53	·011	2	yel, v. bl.
378	3 16·6	58 4	8·2, 9·5	313·0	18·67	·041	2	
392	,, 22·9	52 33	7·5, 9·7	347·2	25·85	·041	2	yel, o.
427	,, 34·5	28 27	6·6, 7·4	208·5	6·70	·085	2	ylsh wh, blsh wh.
430	,, 35·1	4 48	6·0, 9·0	55·4	26·24	·050	2	AB. p. orge, two bl. } Triple
,,	,, ,,	,, ,,	,, 9·8	301·5	37·30	·050	2	AC. } Triple

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
470	3 49·3	− 3 15	4·0, 6·0	347·8	6·75	1916·085	2	32 Erid.; yel, blsh grn.
471	,, 51·1	+39 43	3·1, 8·3	9·6	8·88	·085	2	ε Pers.; grnsh wh, lilac.
485	,, 59·1	62 5	6·1, 6·2	303·1	17·80	·085	2	ylsh wh, ylsh wh.
523	4 13·8	23 30	7·2, 9·2	162·1	10·36	·097	2	wh, bl.
545	,, 21·3	17 59	7·5, 9·3	57·6	18·99	·099	2	v. wh, bl.
552	,, 24·5	39 46	6·3, 6·5	114·0	8·80	·099	2	wh, ylsh wh.
[OΣ 84]	,, 25·7	6 35	6·8, 7·7	255·2	9·56	·087	2	yel, bl.
[,, 87]	,, 30·8	8 1	7·5, 9·2	234·4	6·04	·097	2	wh, blsh.
570	,, 30·5	− 9 56	7·0, 8·0	259·5	12·85	·102	2	ylsh wh, p. bl.
576	,, 33·4	−13 13	7·0, 8·0	172·2	12·27	·100	2	Σ mgs. 6·7, 7·2; Lewis decl. 13°4′.
590	,, 38·8	− 8 59	6·2, 6·7	316·9	8·99	·102	2	55 Erid.; yel, blsh wh.
612	,, 48·9	+ 7 13	7·7, 7·8	197·4	16·44	·099	2	wh, wh.
616	,, 52·5	37 44	5·1, 7·7	354·2	6·09	·099	2	ω Aur.; wh, purp.; Σ mgs. 4·0, 7·9.
645	5 3·5	27 54	6·2, 8·2	26·8	11·85	·134	2	wh, lilac.
646	,, 4·1	39 9	8·2, 9·0	74·0	15·76	·134	2	
653	,, 8·9	32 33	5·0, 7·2	225·6	14·64	·134	2	14 Aur.; p. yel, lilac. AB.
668	,, 9·7	− 8 19	1·0, 8·0	201·6	9·66	·102	2	β Ori.; v. wh, bl.
688	,, 14·7	−10 51	7·0, 7·4	274·3	10·43	·102	2	Lewis decl. −19°51′ p. yel, blsh wh.
692	,, 15·6	− 8 8	7·8, 8·8	3·8	35·12	·102	2	ylsh wh, blsh.
721	,, 24·3	+ 3 4	7·0, 9·0	149·9	24·70	·134	2	A–BC; wh, prpsh.
[OΣ 109]	,, 30·2	71 35	7·7, 9·0	127·8	10·78	·216	2	ylsh, blsh.
785	,, 39·7	25 53	6·7, 7·7	348·1	14·07	·134	2	wh, bl.
792	,, 41·8	− 3 18	8·2, 8·7	132·6	25·00	·134	2	
817	,, 49·4	+ 7 1	8·2, 8·3	72·7	18·42	·134	2	
869	6 5·9	− 9 50	7·5, 8·5	278·5	24·28	·232	2	wh, blsh.
896	,, 17·8	+51 56	8·3, 8·7	82·1	18·70	·232	2	D>.
897	,, 16·2	26 43	8·2, 8·5	348·0	17·83	·232	2	
914	,, 21·9	− 7 25	6·7, 9·0	297·6	20·92	·232	2	blsh wh, bl.
[OΣ 143]	,, 25·3	+17 1	6·8, 9·9	101·6	8·03	·216	2	ylsh, o.
924	,, 26·0	17 51	6·0, 6·9	210·0	20·03	·216	2	20 Gem.; ylsh wh, grnsh wh.
960	,, 41·6	53 9	7·3, 9·2	66·8	21·75	·243	2	P. VI 215; wh, prpsh.
1010	,, 56·4	− 2 58	7·8, 8·8	5·6	23·11	·232	2	wh, blsh.
1069	7 13·6	−13 31	8·3, 8·3	14·9	25·26	·247	2	A<s?
1082	,, 18·3	+10 54	8·5, 9·0	326·7	19·92	·247	2	Σ 8·0, 8·7—but mgs. as Lewis.
1111	,, 27·0	− 8 28	8·2, 8·7	219·8	20·00	·247	2	

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
[OΣ 173 *rej.*]	7 27·9	+33 20	Fr.7·0, 9·5	348·5	43″67	1916·247	2	AC. OΣ 7·5, 9·0 wh, bl.
1138	,, 40·9	−14 27	6·2, 7·0	339·5	16·62	·247	2	2 Argûs; wh, lilac.
1141	,, 41·9	+ 0 15	8·0, 8·7	8·9	17·73	·247	2	
1188	8 3·2	30 38	8·0, 8·5	201·5	16·42	·254	2	D<s?
1193	,, 10·0	72 44	6·0, 9·0	85·7	43·47	·254	2	yel, bl. D>s?
1232	,, 26·3	66 38	8·0, 8·2	350·6	31·50	·254	2	
1254	,, 34·6	+20 2	Fr.6·5, 9·5	53·5	20·10	·254	2	p. or, bl; Lewis 5·7 9·8. P. VIII 29. In *Præsepe.*
1266	,, 38·3	28 49	8·0, 9·2	65·1	23·60	·254	2	A and D< ?
[OΣ 195]	,, 48·6	8 48	7·4, 7·9	138·6	9·50	·254	2	P. VIII 200; ylsh wh, blsh wh.
1311	9 1·7	23 21	6·7, 7·0	200·1	7·17	·265	2	wh, wh.
1327	,, 9·6	28 20	8·0, 9·2	73·4	10·84	·265	2	AB. A and D >.
,,	,, ,,	,, ,,	,, 9·1	21·7	26·75	·265	2	AC. A >. D <. Triple.
1364	,, 26·1	20 28	7·7, 9·2	155·6	16·30	·265	2	wh, blsh.
1416	10 7·5	−15 35	7·4, 8·6	277·4	11·43	·265	2	Σ has 6·7 mg; but overrated.
1427	,, 16·0	+44 25	7·2, 7·7	214·3	9·54	·278	2	wh, blsh wh.
1440	,, 24·4	− 3 23	8·0, 9·5	347·6	15·29	·294	2	
1473	,, 42·7	−15 6	8·0, 8·9	9·9	30·73	·268	2	P. X 159.
1474	,, 42·7	−14 44	6·9, 8·0	23·1	69·55	·265	2	AB. D >; A is moving. All white.
,,	,, ,,	,, ,,	8·0, 8·0	197·5	6·65	·265	2	CB. (not BC, as in Lewis). Triple.
1509	11 1·5	−12 52	7·2, 9·0	16·1	32·93	·294	2	p. yel, bl.
1529	,, 14·3	− 1 6	7·0, 8·0	251·3	9·89	·278	2	ylsh wh, blsh wh.
[OΣ 239 *rej.*]	,, 39·0	+25 46	5·5, 9·8	23·3	38·07	·294	2	yel, blsh; A<s.
1570	,, 40·3	46 10	8·3, 8·8	48·8	10·70	·308	2	
1595	,, 59·2	7 57	8·3, 9·0	329·7	27·63	·308	2	
1605	12 5·3	− 1 41	8·0, 8·5	278·4	23·44	·308	2	
1608	,, 6·5	+53 59	7·5, 7·7	223·1	11·35	·315	2	wh, wh.
1635	,, 16·0	−10 55	7·7, 8·7	173·4	13·33	·313	2	wh, wh.
1645	,, 23·2	+45 21	7·0, 7·5	160·2	10·27	·315	2	ylsh wh, ylsh wh.
1649	,, 26·5	−10 31	7·5, 7·8	194·3	15·43	·313	2	ylsh wh, blsh wh.
1670	,, 36·6	− 0 54	3·0, 3·0	324·0	6·00	·315	3	γ Virg; both yelsh wh.
1675	,, 39·6	+34 58	8·3, 9·0	7·4	30·52	·321	2	A>s?
1691	,, 50·7	58 42	8·2, 9·0	276·5	18·83	·321	2	
1701	,, 54·2	7 2	7·5, 9·5	305·9	21·53	·321	2	yel, blsh.

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
1719	13 2·2	+ 1 8	7·3, 7·8	360·2	7·16	1916·335	2	A > s; ylsh wh, ylsh wh.
1724	,, 4·8	− 5 0	4·0, 9·0	343·2	7·09	·335	2	AB. θ Virg.; wh, prp, o. AC. A and D<. Triple.
,,	,, ,,	,, ,,	,, 10·0	299·2	72·36	·335	2	
1741	,, 19·1	− 1 35	8·2, 9·7	261·9	25·58	·343	2	D<s.
1775	,, 38·3	− 3 46	7·0, 9·7	335·6	27·75	·343	2	P. XIII 171; p. orge, o.
1791	,, 52·0	+14 56	8·7, 9·5	159·9	21·02	·343	2	
1803	14 2·3	38 54	7·7 9·5	42·7	17·88	·370	2	ylsh wh, blsh.
1827	,, 11·1	59 42	8·5, 9·0	209·3	11·28	·370	2	A>s.
1835	,, 18·5	8 53	5·5, 6·8	191·0	6·39	·371	3	P. XIV 69; wh, lilac; A<.
1864	,, 36·0	16 50	4·9, 6·0	103·7	5·91	·371	3	π Boöt.; grnsh wh, prpsh wh; A<.
1899	,, 56·4	− 2 43	7·2, 9·7	67·3	28·25	·370	2	yel, blsh.
1914	15 6·5	− 5 6	8·0, 8·7	335·5	31·12	1915·492	2	
1920	,, 7·5	+47 13	8·5, 8·5	290·8	18·96	1916·376	2	
[OΣ 292 *rej.*]	,, 10·0	32 10	6·5, 8·0	155·6	124·75	1915·492	2	yel, bl; D>?
1936	,, 16·9	27 22	8·5, 9·0	231·9	20·19	1916·373	2	
1939	,, 22·1	−10 37	8·0, 9·0	131·2	8·69	·373	2	D>s?
1948	,, 24·0	+55 14	8·0, 8·7	48·8	12·90	1915·495	2	A>s, D<s.
1977	,, 45·3	25 46	7·7, 9·7	357·2	13·74	·622	2	ylsh, o.
1978	,, 46·3	14 58	8·5, 9·0	235·0	15·27	1916·373	2	
1996	,, 54·4	57 33	8·7, 9·0	109·2	19·70	·376	2	D < s.
[OΣ 305]	16 7·8	33 36	5·8, 9·8	263·4	5·45	·386	2	p. yel, o.
2019 *rej.*	,, 8·8	−10 10 Fr.	8·0, 9·8	154·1	22·33	1915·503	3	ylsh, o.
2033	,, 13·0	− 2 2	8·5, 8·7	174·8	10·64	1916·373	2	
[OΣ 312]	,, 22·6	+61 44	2·1, 8·1	142·1	5·12	·386	2	η Drac.; yel, bl.
2071	,, 34·2	13 52	8·7, 9·0	311·1	24·93	·373	2	
2083	,, 38·1	13 48	8·3, 8·8	334·3	12·56	·373	2	
[OΣ 317]	,, 49·9	44 34	7·2, 8·0	317·5	118·90	1915·492	2	AC. ylsh wh, blsh wh; D<.
2124	,, 58·9	65 21	8·5, 9·2	88·1	15·02	1916·386	2	
2128	17 1·9	59 43	8·0, 9·2	49·6	12·36	1915·505	2	A > D<.
2140	,, 10·1	14 30	3·0, 6·1	114·5	4·86	·608	3	α Herc.; orge, blsh green.
2144 *rej.*	,, 11·9	− 7 45 Fr.	8·0, 9·5	183·2	25·38	·508	2	Lewis decl. 7° 39′; yel, o.
2155	,, 14·9	+60 48	6·2, 9·5	111·1	9·96	1916·386	2	ylsh wh, blsh; A > D<s.
2158	,, 19·1	3 10	8·0, 9·7	78·2	23·31	1915·622	2	
2190	,, 31·7	21 3	6·0, 9·5	23·2	10·10	·606	2	P. XVII 163; wh, prpsh.
2211	,, 41·5	− 1 10	8·2, 9·2	115·8	10·55	·508	2	D<s.

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.		Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′			°	″			
2216 *rej.*	17 42·1	+ 5 43	Fr. 8·0,	9·5	31·3	28·16	1916·508	2	wh, bl (not in Lewis).
2217	,, 42·1	14 49	8·4,	7·8	284·1	6·78	1915·608	3	wh, wh.
[OΣ 336 *rej.*]	,, 44·2	34 19	6·3,	10·3	163·7	42·40	·518	2	p. yel, o.; Hussey decl. 35° 19′.
2227	,, 44·6	5 23	8·8,	8·8	110·7	17·88	1916·508	2	A and D>.
2232	,, 46·2	25 18	7·0,	8·5	142·1	6·50	1915·633	2	wh, blsh.
2234	,, 47·3	− 7 55	8·6,	9·3	198·9	17·23	1916·538	2	D<s.
2247	,, 55·6	+29 30	8·5,	9·0	189·7	11·52	·532	2	
2250	,, 53·9	− 6 51	8·0,	9·0	346·4	7·85	·609	2	
2261	,, 55·8	+52 14	7·5,	9·5	261·5	9·48	·532	2	ylsh wh, blsh.
2263	,, 56·9	26 33	8·2,	9·2	160·6	7·47	·609	2	
2265	,, 59·2	6 28	8·4,	9·4	282·2	24·36	1915·508	2	
2291	18 6·6	34 1	8·5,	9·0	159·3	27·20	1916·538	2	D<.
[OΣ 346]	,, 11·1	19 44	7·5,	8·3	328·5	5·60	1915·518	2	ylsh wh, blsh wh.
2301	,, 11·6	23 58	8·5,	9·0	123·6	21·66	1916·544	2	D>.
2313	,, 19·3	− 6 39	7·2,	8·5	198·5	6·09	1915·634	2	wh, blsh.
2333	,, 27·4	+32 10	7·5,	8·1	333·9	6·50	·610	2	wh, blsh wh.
2337	,, 29·2	−14 48	7·8,	8·8	297·3	16·29	·527	2	P. XVIII 116; ylsh wh, bl.
2351	,, 33·0	+41 12	7·4,	7·4	339·9	5·06	·634	2	D>s? wh, wh.
2361	,, 35·6	3 1	8·3,	8·8	211·8	24·97	·537	2	
2371	,, 38·3	27 32	8·5,	8·5	56·2	9·61	1916·609	2	
2378	,, 39·8	35 28	8·3,	8·8	190·0	11·13	1915·541	2	Lewis decl. 35° 7′.
2411	,, 47·7	14 25	7·0,	9·8	95·8	13·36	·622	2	ylsh wh, o.
2413	,, 48·4	3 15	8·2,	8·7	200·3	10·01	1916·544	2	Lewis R.A. 43^{m}·4.
2421	,, 52·3	33 39	8·0,	8·7	62·3	23·02	1915·537	2	
2427	,, 54·6	38 5	8·5,	9·0	61·0	49·03	1916·547	2	AB. } Triple.
,,	,, ,,	,, ,,	9·0,	9·2	80·2	7·20	·547	2	BC. }
2452	,, 56·9	75 39	6·7,	7·5	218·6	5·76	·532	2	wh, lilac.
2459	19 3·2	25 49	8·4,	9·1	232·9	13·67	1915·568	2	
2489	,, 11·9	14 22	6·5,	9·5	347·1	8·23	1916·547	2	wh, bl.
2494 *rej.*	,, 14·6	− 6 49	Fr. 7·0,	10·0	82·2	25·68	1915·562	2	=Krüger 1544 (OR). ruddy orge, bl (fine cols.).
2505	,, 16·2	+35 22	8·0,	8·7	315·2	10·20	1916·547	2	
2506 *rej.*	,, 17·2	14 10	8·7,	9·1	351·2	16·74	·570	2	wh, wh.
2508	,, 14·4	67 44	8·7,	9·0	116·7	17·70	·570	2	
2512	,, 18·8	31 32	7·5,	9·8	310·6	22·08	1915·622	2	ylsh wh, o.
2523	,, 22·5	20 57	7·3,	7·4	148·7	6·18	·610	2	wh, wh.
2531	,, 24·5	2 53	7·8,	9·7	30·4	31·82	·601	2	wh, bl.
2548	,, 32·3	24 46	8·0,	9·0	100·2	9·20	1916·570	2	

Measures of Double Stars—continued.

Σ.	R.A. 1900.	Decl. 1900.	Magnitudes.		Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′			°	″			
2585	19 44·5	18 54	+5·7,	8·8	311·7	8·71	1916·578	2	AB–C. ζ Sagittæ; wh, bl.
[OΣ 388]	,, 48·1	25 36	7·6,	8·8	134·7	27·14	1915·518	2	BC. (AB 7·6, 7·6: 3″·97).
,,	,, ,,	,, ,,	7·6,	,,	135·8	31·11	·518	2	AC. 2 ylsh wh, 1 bl. } Triple.
2621	,, 59·7	8 57	7·7,	7·9	223·1	5·65	·633	2	wh, wh.
2648	20 7·4	49 31	7·9,	9·2	117·1	6·33	1916·578	2	Lewis 8·2 mag. (B); ylsh wh, blsh.
2655	,, 9·7	21 55	7·5,	7·5	2·8	6·10	1915·610	2	wh, wh.
2667	,, 14·2	45 19	8·2,	8·5	224·3	7·99	1916·578	2	
2669	,, 17·0	55 47	8·7,	9·1	260·2	23·43	·578	2	
2675	,, 12·3	77 25	4·5,	8·0	123·3	7·38	1915·555	3	κ Ceph.; wh, bl.
2683	,, 22·9	−13 30	8·0,	8·5	67·1	22·82	·568	2	
2699	,, 31·4	−13 5	8·0,	9·0	195·1	9·58	1916·601	2	
[OΣ 409]	,, 35·3	+ 3 5	6·8,	10·3	84·0	17·13	1915·568	2	AB. wh, o.
,,	,, ,,	,, ,,	,,	8·0	333·9	65·30	·568	2	AC. ,, bl. } Triple.
2718	,, 37·8	12 22	7·4,	7·6	86·6	8·33	·572	2	wh, wh.
2722	,, 39·1	19 22	8·2,	8·7	306·9	7·18	1916·578	2	
2726	,, 41·5	30 21	4·0,	9·2	65·6	6·04	1915·625	2	52 Cyg.; or yel, bl; A<D>.
[OΣ 413]	,, 43·5	36 7	4·6,	9·0	105·6	84·75	·552	2	λ Cyg. AB–C; ylsh wh, bl; A<D>s.
[,, 416]	,, 48·4	43 22	7·8,	8·1	130·9	7·67	·572	2	wh, wh. A>D<s.
2759	21 2·3	32 3	8·5,	9·5	326·6	17·05	1916·585	2	A and D<.
2760	,, 2·7	33 44	7·3,	8·1	228·7	5·20	1915·634	2	A<D>; wh, blsh.
2763	,, 4·8	16 56	8·5,	9·7	292·9	17·50	1916·585	2	D<.
2789	,, 16·8	52 33	7·1,	7·1	116·3	6·00	1915·625	2	wh, wh.
3112	,, 29·6	9 3	7·6,	9·4	239·2	7·14	·612	2	=OΣ 528; ylsh wh, o.
2813	,, 32·9	57 2	8·5,	9·0	272·9	10·35	1916·585	2	
[OΣ 443]	,, 32·6	6 16	8·0,	8·3	349·1	8·16	1915·612	2	wh, wh.
[,, 447]	,, 35·6	41 16	7·0,	7·9	45·2	29·01	·616	2	yel, ylsh.
2816	,, 35·8	57 2	6·3,	7·9	120·4	11·66	·616	2	AC. P. XXI 248.
,,	,, ,,	,, ,,	,,	8·0	339·5	19·95	·616	2	AD. ylsh wh, 2 blsh. } Triple.
2817	,, 37·3	− 0 2	8·2,	8·5	156·3	25·84	·616	2	
2828	,, 44·5	+ 2 56	8·0,	9·0	142·3	28·24	·616	2	AB. } Triple.
,,	,, ,,	,, ,,	,,	9·2	133·3	27·85	·616	2	AC.
[OΣ 454]	,, 51·5	23 52	7·0,	9·0	279·2	7·13	·616	2	wh, blsh.
[,, 455]	,, 51·9	15 39	7·5,	9·0	269·3	9·88	·616	2	ylsh, blsh.
2851	,, 56·3	−12 28	8·0,	8·3	121·3	18·32	·628	2	
2855	22 0·2	− 1 55	7·9,	9·5	299·9	26·93	·628	2	wh, o.
2860	,, 0·0	+60 21	7·7,	9·3	256·0	7·56	1916·585	2	D<; yel, bl.

Measures of Double Stars—continued.

Σ	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
[OΣ 461]	22 1·0	+59 23	5·9, 10·6	298·3	11·18	1916·585	2	AB. 15 Ceph. } Triple.
,,	,, ,,	,, ,,	,, 9·5	39·3	89·51	·585	2	AC. yel, o, o. }
2861	,, 1·4	20 18	7·7, 8·2	220·1	6·99	1915·631	2	wh, wh.
2863	,, 0·9	64 8	4·7, 6·5	281·1	7·18	·620	2	ξ Ceph. ; ylsh wh, ruddy lilac.
2877	,, 9·5	16 42	6·4, 9·6	7·2	12·97	·620	2	P. XXII 33 ; p. or, grnsh.
2891	,, 12·4	47 29	8·2, 9·2	309·2	13·00	·620	2	
2902	,, 19·3	44 51	7·1, 8·0	89·2	6·37	·625	2	yel, grnsh.
2908	,, 23·3	16 45	7·0, 8·7	115·1	8·88	·625	2	p. yel, blsh.
2913	,, 25·2	− 8 38 Fr.	7·7, 8·7	329·9	8·16	1916·777	2	Σ mags. 7·0, 8·0.
2922	,, 31·4	+39 7	6·0, 6·5	185·7	22·38	·590	2	AB. 8 Lacertæ ; ylsh wh, blsh wh. } Quad.
,,	,, ,,	,, ,,	6·5, 10·2	155·3	28·06	·590	2	BC. blsh wh, o.
,,	,, ,,	,, ,,	,, 8·5	131·4	66·72	·590	2	BD. ,, bl. }
2923	,, 30·4	69 51	6·9, 9·2	46·7	9·33	1915·620	2	creamy wh, prpsh.
2926	,, 33·0	38 23	8·5, 8·5	335·8	21·11	1916·590	2	
2930	,, 34·4	6 38	8·3, 9·3	77·2	21·42	·595	2	
2932	,, 36·8	29 30	8·7, 9·2	281·3	20·44	·595	2	D<s.
2938	,, 39·9	− 3 11	8·2, 8·2	162·8	19·47	·777	2	
[OΣ 529]	,, 41·7	+67 36	8·5, 9·2	203·5	3·58	·590	2	AB. ylsh wh, blsh.. } Triple.
,,	,, ,,	,, ,,	,, 9·0	219·2	20·68	·590	2	AC. ylsh wh. blsh. }
2952 *rej.*	,, 49·4	27 29	7·5, 10·0	135·7	18·24	1915·624	2	ylsh wh, o.
2953	,, 48·7	60 24	6·9, 9·4	137·9	8·40	1916·590	2	yel, blue.
2954	,, 49·8	14 40	9·0, 9·0	23·8	39·65	·601	2	A>D<.
[OΣ 536]	,, 53·5	8 50	7·0, 10·0 (Fr.)	84·1	238·31	1915·665	2	AB–C ; yel, o. ; A and D>.
2978	23 2·7	32 17	6·8, 8·0	145·3	8·43	·625	2	P. XXII 306 ; wh, p. bl.
2991 *rej.*	,, 8·4	10 31	6·0, 10·0	358·2	33·03	·631	2	yel, o. (not in Lewis).
2993	,, 8·8	− 9 28	7·0, 7·8	177·2	25·57	·665	2	AB. ylsh wh, wh. } Triple.
,,	,, ,,	,, ,,	,, (Fr.) 8·5	116·6	120·96	·665	2	AC. ylsh wh, blsh wh. }
2997	,, 12·1	+20 53	8·5, 9·0	222·4	24·39	1916·794	2	
3004	,, 16·0	43 35	6·5, 10·0	178·1	13·07	·794	2	wh, o.
3006	,, 16·4	34 54	8·5, 9·0	161·0	5·67	·801	2	A>.
3007	,, 17·8	20 1	6·5, 9·5	82·8	6·28	·801	2	wh, o.
3009	,, 19·2	3 11	6·8, 8·8	228·1	7·02	1915·647	2	orge yel, bl.
3010	,, 18·7	45 15	8·0, 8·7	132·1	25·76	·640	2	
3021	,, 26·3	15 40	7·7, 8·9	306·3	8·45	·647	2	ylsh wh, blsh.

Measures of Double Stars—continued.

Σ.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nts.	Remarks.
[OΣ 501]	23 35·1	+37 6	6·8, 10·2	164·3	14·99	1916·794	2	p. yel, o.
[,, 504]	,, 37·4	18 7	7·2, 10·0	175·3	7·70	·794	2	yel, blsh.
3039	,, 41·8	27 52	7·3, 9·7	32·4	33·80	1915·647	2	yel, o.
[OΣ 509]	,, 45·5	42 52	7·6, 9·5	106·1	5·50	1916·801	2	wh, o.
3051	,, 57·6	79 43	7·5, 9·4	23·4	16·72	1915·640	2	ylsh wh, o.
[OΣ 514]	,, 59·5	41 32	6·9, 9·5	168·5	5·31	1916·801	2	wh, o.
[OΣ 547]	0 0·4	45 16	Fr.8·3, 8·5	141·2	4·70	·801	2	both p. orge; A<; mgs. underrated.
3061	,, 0·6	17 17	8·0, 8·0	147·5	7·66	·801	2	
Hh 503	16 14·3	−19 19	7·5, 8·0	332·6	47·05	·501	2	P. XVI 44-5; ylsh wh, blsh wh.
,, 504	,, 18·8	−19 53	8·0, 9·0	22·3	12·72	·501	2	P. XVI 48-9; wh, greyish.
,, 505	,, 15·1	−25 21	3·1, 8·0	271·6	20·34	·501	2	σ Scorpii; grnsh wh, prpsh.
,, 507	,, 18·4	−29 29	5·9, 6·6	355·6	7·00	·501	2	p. yel, blue (very pretty).
,, 522	,, 38·7	+ 6 48	7·5, 8·0	291·1	54·00	·480	2	ylsh wh, blue.
,, 523	,, 41·0	8 46	5·4, 8·0	230·3	81·34	·480	2	43 Herc.; p. orge, blue.
,, 562	18 9·4	28 14	7·7, 7·9	136·7	56·50	·508	2	wh, wh; D<(β 54·9: 1880).
Sh 263	,, 11·1	−18 49	7·0, 8·5	12·8	54·39	·598	2	wh, blue.
,, 264	,, 12·8	−18 40	7·0, 7·7	52·3	17·18	·598	2	ylsh wh, lilac.
Hh 605	19 10·0	+38 52	8·0, 8·3	56·7	39·34	·566	2	wh, wh.
,, 608	,, 12·9	37 57	4·5, 9·0	71·2	100·24	·516	2	θ Lyræ; yel, blue.
,, 611	,, 15·8	−19 25	6·4, 9·0	309·2	90·53	·516	2	P XIX 67; ylsh wh, o.
"Cel. Obj."	,, 33·5	−17 8	7·0, 7·5	237·5	10·60	·516	2	wh, wh; *sp* 54 Sagittarii.
Hh 630	,, 35·0	+16 23	7·0, 9·0	303·0	28·18	·550	2	orge, blue (v. fine cols).
"Cel. Obj."	,, 38·2	1 0	8·5, 9·0	18·4	27·95	·556	2	wh, blsh.
S 738	20 6·1	33 20	7·1, 7·7	109·7	41·60	·582	2	wh, wh.
,, 752	,, 25·7	19 5	7·0, 7·2	288·5	106·10	·582	2	ylsh wh, blsh wh.
"Cel. Obj."	,, 29·0	28 31	8·0, 9·0	339·7	33·96	·578	2	AB. wh, blsh. } Triple.
,,	,, ,,	,, ,,	,, 9·5	223·6	36·90	·578	2	AC. ,, blsh. }
Hh 706	,, 42·6	−18 34	7·0, 7·1	293·6	15·85	·570	2	wh, wh,
,, 753	22 8·8	−21 35	5·5, 8·0	113·6	5·27	·777	2	AB. A> D<; orge yel, blue. 41 Aquarii. } Triple.
,, ,,	,, ,,	,, ,,	,, 9·0	43·5	211·81	·777	2	AC. orge yel, blue. }
,, 762	,, 21·2	−17 15	6·4, 6·6	312·3	6·96	·777	2	53 Aquarii; both ylsh wh; A<D>.
,, 807	23 40·8	−19 14	5·5, 6·5	137·8	6·18	·787	2	107 Aquarii; ylsh wh, lilac; A>D<.

Notes on some doubtful Hh Objects.

Hh 491. Is pretty certainly identical with Σ 1999 (Hh 492), with an error of 14′ in decl.

550. Not found; so Innes (*Transvaal Obs. Circular*, No. 1). But it bears a suspicious resemblance to Hh 552, which is 1° 14′ to the south. Query identical?

612. There is little doubt that this is Σ 2531, with 2° error in R.A. It is the *f* of two stars (not the *p*, as J. H. has it) about 20′ apart, forming a right angle with a wider pair about 25′ *n*.

615. Probably identical with Σ 2533; no other corresponding object near the place.

659. Not 64 Sagittarii. But it may be identical with Σ 2625 (P. XIX 396) which is *sf* 65 Sagittarii? A similar mistake occurred with 64 Draconis (Hh 664), afterwards corrected to 65.

670. Not found. In a rich part of the Milky Way; the vague description might apply to several stars.

672. Strong probability that this is identical with Hh 671 (P. XX 12), 42′ *n* of this place.

715. Burnham identifies this with β 269; which seems unlikely, as it is much too unequal and difficult for one of H's close pairs. It is more probably identical with Σ 2742 (λ Equulei), which fits it better, and is quite as near to the place given.

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Sir W. Herschel's Double Star Measures. By W. S. Franks.

When going through the Synoptic Catalogue of H's double stars (vol. xxxv., *Memoirs*) recently, in connection with my working list, I was struck with the pretty constant difference between H's distance measures and my own (the latter being almost invariably in excess) of certain *wide* double stars. I then consulted Burnham's measures of some of these objects, and was astonished to find the same peculiarity—his average deviation being even greater than mine. Pursuing the matter further, I made a comparison between Herschel and Struve, taking all measured objects common to both from the *Mensuræ Micrometricæ* (Lewis's revision and Lord Lindsay's summary)—except those of H's Class I., which are more like estimates than measures. The result was a confirmation of the previously found discrepancy, as will be seen by the following epitome:—

I. *Comparison between* H *and* Σ.

H's Class.	Limiting Distance.	No. of Stars compared.	Mean Distance.	Mean Difference.	Percentage.
II.	4″ to 8″	18	5″·6	−0″·27	−4·79
III.	8 ,, 16	52	11·2	−0·20	−1·79
IV.	16 ,, 32	60	21·1	−1·07	−5·08
V.	32 ,, 60	15	41·7	−3·56	−8·54

II. *Comparison between* H *and* β.

H's Class.	Limiting Distance.	No. of Stars compared.	Mean Distance.	Mean Difference.	Percentage.
IV.	16″ to 32″	11	21″·8	−1″·11	−5·09
V.	32 ,, 60	33	44·7	−3·78	−8·46
VI.	above 60	19	106·8	−9·01	−8·43

III. *Comparison between* H *and* Fr.

IV.	16 to 32	6	22·8	−0·95	−4·17
V.	32 ,, 60	27	42·0	−3·35	−7·93
VI.	above 60	21	78·3	−4·85	−6·20

IV. *Combined Results.*

					H *minus*
IV.	16 to 32	77	21·9	−1·04	−4·75	Σ, β, and Fr.
V.	32 ,, 60	75	42·8	−3·56	−8·31	do.
VI.	above 60	40	92·5	−6·93	−7·31	β and Fr.

No duplicates are included in the three lists, which are composed of entirely different stars. Any objects showing definite evidence of motion, as in Σ's Appendix II., were rejected. It may be mentioned that there are complete measures for only about half the stars in the Hh list; or, excluding duplicates in different classes, about 370 out of 812.

It will be noticed that H's distance measures show the smallest error for Class III., and increase in nearly equal ratio from that on either side. Probably this was because those doubles were easier and more certain to measure than either the closer or wider ones. Considering, however, the very rough and crude appliances at Herschel's command, and the absence of a driving-clock, the error is no more than might have been expected. On the whole, his distance measures require a *plus* correction of 5 to 8½ per cent., from Class IV. outward, to make them comparable with those by other observers. It seems necessary to emphasise this point, as I have come across some instances in which H's measures of wide double stars were quoted in conjunction with quite modern ones, with about a century's interval, and the difference deduced as proper motion—an assumption which is not justified by any means. Despite the temptation to utilise what, in many cases, are the earliest recorded measures, it is certainly not safe to depend upon them as evidence of motion, unless corroborated by subsequent observations.

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B.D. Stars observed as Missing or Faint. By Rev. T. E. Espin, M.A.

In the course of the systematic examination of the stars of the B.D. for new pairs, some have been found to be altogether missing, others were so faint that it seems almost impossible that they could have been observed by Argelander unless they were at that time brighter than they are now. Where a star is missing, this may arise from (1) an error in the original observation, (2) variation, (3) proper motion. As far as I am aware, none of the stars in this list are known variables.

Column 1 is a current number; columns 2, 3, 4, 5 give the number, place, and magnitude in the B.D.; column 6, the observed magnitude, if seen; the symbol < is employed to indicate the faintest star that would be seen at the time; column 7, the date of observation; column 8, the apparent magnitude on the Harvard map of the sky (section, Decl. +45°). In the notes will be found additional observations, and any stars near. Lists of other missing B.D. stars are given in Chandler's "Third Catalogue of Variable Stars," and in Harvard Circular No. 128.

No.	B.D.	R.A. 1855. h m s	Decl. 1855. ° ′	Mag.	Obs.	Date.	H.M.
1	+45° 92	0 18 49·6	+45 23·8	9·5	12	14 Jan. 6	11
2	+43° 120	29 32·0	43 59·3	9·5	...	15 Jan. 16	
3	+47° 192	38 2·4	47 11·6	9·2	...	13 Nov. 17	
4	+49° 327	1 5 1·1	49 34·2	9·5	12	12 Jan. 5	
5	+48° 441	19 18·0	48 26·5	9·5	<13·5	13 Nov. 5	10
6	+47° 439	22 52·4	47 24·2	9·2	9·5	13 Dec. 1	10·5
7	+49° 525	52 7·1	49 33·5	9·5	...	12 Jan. 27	
8	+44° 542	2 27 52·8	44 51·7	9·5	<14	14 Nov. 26	
9	+45° 639	28 20·3	45 54·3	9·4	13	13 Dec. 23	?
10	+44° 551	30 20·6	44 57·1	9·4	...	14 Jan. 6	
11	+43° 632	57 9·6	43 9·3	9·5	12·5	15 Dec. 8	
12	+48° 869	3 4 40·4	48 30·5	9·5	<12	12 Jan. 27	?
13	+44° 799	38 50·1	44 47·9	9·3	...	15 Jan. 16	
14	+49° 1052	43 2·3	49 45·5	9·5	<13	11 Dec. 20	
15	+50° 968	4 6 59·0	50 56·6	9·5	...	11 Jan. 20	
16	+43° 1105	42 58·4	43 45·3	9·5	13·5	15 Feb. 20	
17	+44° 1216	5 19 22·8	44 11·6	9·5	<13·5	16 Feb. 7	
18	+50° 1344	6 31 58·7	50 42·2	9·5	13	1 Jan. 13	?
19	+42° 1656	55 48·1	42 6·0	9·5	...	16 Mar. 31	
20	+42° 1683	7 4 54·2	42 5·8	9·4	...	15 Apr. 7	
21	+46° 1267	16 24·6	46 28·2	9·5	...	13 Apr. 4	
22	+42° 1776	43 36·6	42 49·8	9·5	...	15 Mar. 30	?
23	+41° 1778	53 58·0	41 20·6	9·5	12	16 Apr. 2	10
24	+46° 1365	8 1 14·3	46 49·5	9·3	...	12 Mar. 28	

No.	B.D.	R.A. 1855.	Decl. 1855.	Mag.	Obs.	Date.	H.M.
		h m s	° ′				
25	+44° 1727	8 5 22·5	+44 44·2	9·5	...	13 Apr. 4	11
26	+46° 1408	20 14·5	46 53·8	9·3	11·8	12 Mar. 28	11
27	+43° 1850	35 28·1	43 59·9	9·5	13	13 Apr. 5	11
28	+41° 1894	49 35·1	41 44·9	9·5	13·5	16 Apr. 4	
29	+46° 1465	52 45·6	46 16·9	9·3	<12	12 Mar. 28	
30	+42° 1966	57 14·3	42 11·2	9·5	12·5	16 Apr. 1	
31	+41° 1939	9 6 51·5	41 33·9	9·5	...	16 Apr. 2	
32	+43° 1939	29 12·9	43 21·4	9·5	...	13 Mar. 25	
33	+43° 1946	30 44·3	43 6·8	9·5	<14	13 Apr. 5	
34	+42° 2037	32 26·9	42 14·6	9·4	...	16 Apr. 1	
35	+46° 1553	40 50·5	46 28·2	9·3	...	12 Mar. 20	
36	+53° 1379	52 24·3	53 34·1	9·5	...	09 Apr. 7	?
37	+46° 1585	58 38·9	46 50·6	9·5	11·5	12 Apr. 10	11
38	+46° 1597	10 2 54·8	46 52·5	9·5	...	12 Apr. 2	
39	+40° 2302	4 19·8	40 45·2	9·5	...	16 Apr. 4	
40	+43° 2010	13 39·5	43 3·0	9·5	...	15 Mar. 31	
41	+41° 2083	15 50·6	41 34·4	9·5	...	15 May 2	
42	+45° 1858	35 19·3	45 37·1	9·5	<13	12 Apr. 10	
43	+41° 2149	52 7·4	41 16·0	9·5	...	15 Apr. 29	
44	+45° 1893	58 31·4	45 9·8	9·5	<13·5	12 Apr. 11	
45	+43° 2120	11 21 32·4	43 3·0	9·5	...	13 Apr. 19	
46	+40° 2522	12 11 23·7	40 50·0	9·5	...	15 May 12	
47	+43° 2268	45 55·7	43 8·8	9·5	13	13 May 2	
48	+43° 2305	13 4 39·2	43 20·5	9·5	...	13 May 18	
49	+44° 2267	14 51·9	44 43·0	9·3	...	12 May 13	
50	+42° 2404	23 27·5	42 48·7	9·5	12	13 May 24	
51	+40° 2707	49 46·7	40 12·9	9·5	...	16 May 3	?
52	+42° 2460	56 24·9	42 10·8	9·5	...	13 May 25	
53	+43° 2546	15 53 31·2	43 26·7	9·5	...	16 May 26	?
54	+41° 2736	16 35 48·7	41 21·5	9·5	...	16 July 27	
55	+42° 2861	17 25 11·1	42 7·4	9·5	...	16 July 27	
56	+43° 2745	26 12·5	43 49·2	9·5	<13·5	15 Sept. 10	
57	+41° 2862	31 57·0	41 55·0	9·5	...	16 July 28	
58	+40° 3183	33 1·5	40 40·7	9·5	...	16 Aug. 8	
59	+43° 2845	51 46·5	43 9·1	9·1	...	15 Sept. 10	
60	+41° 3166	18 47 23·6	41 54·5	9·5	...	16 Aug. 2	
61	+42° 3228	57 26·9	42 6·2	9·5	11·5	16 July 28	
62	+41° 3230	19 1 4·0	41 54·5	9·5	...	16 Sept. 24	
63	+43° 3180	7 54·3	43 44·3	9·5	13	15 Aug. 26	10
64	+43° 3218	15 52·6	43 18·6	9·5	...	15 Sept. 10	
65	+41° 3348	22 4·9	41 34·0	9·5	<12	16 Oct. 26	

No.	B.D.	R.A. 1855. h m s	Decl. 1855. ° ′	Mag.	Obs.	Date.	H.M.
66	+43° 3493	20 2 35·3	+43 40·9	9·5	...	15 Aug. 25	?
67	+48° 3072	11 38·3	48 37·7	9·5	...	11 Sept. 16	
68	+42° 3742	20 52·0	42 27·3	9·5	...	16 Sept. 1	
69	+45° 3251	38 45·2	45 3·4	9·5	...	12 Oct. 14	
70	+42° 3991	21 4 49·3	42 55·1	9·5	<14	15 Sept. 9	
71	+45° 3707	46 25·6	45 43·4	9·3	13	13 Nov. 5	11
72	+47° 3606	49 29·0	47 1·2	9·5	...	12 Oct. 14	
73	+41° 4340	53 52·2	41 4·5	9·5	...	16 Sept. 23	
74	+42° 4378	22 17 47·0	42 1·7	9·5	11	16 Sept. 5	11
75	+46° 3811	38 33·5	46 4·8	9·5	13	13 Nov. 3	
76	+46° 3867	47 12·5	46 18·3	9·5	...	13 Oct. 21	
77	+49° 4032	58 27·3	49 18·8	9·5	13	12 Oct. 4	?
78	+45° 4193	23 12 24·0	45 58·2	9·1	9·5	13 Oct. 9	9·5
79	+45° 4202	14 48·5	45 59·6	9·5	12	13 Oct. 9	11
80	+46° 4067	24 8·9	46 2·3	9·5	13·5	13 Oct. 9	
81	+45° 4266	27 26·9	45 56·8	9·5	13·5	13 Oct. 21	?

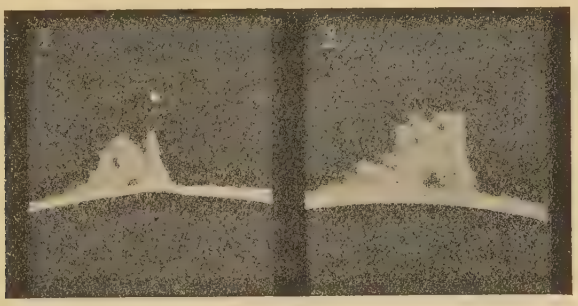
Notes.

2. A star 2′ N.
3. Dec. 31. No star here.
8. 15 Jan. 16. No star.
10. A star 2′ N.
12. H.C. No. 128 missing. The star is apparently visible on H.M.
13. A star 1^m *f*. (This is corrected in Argelander, vol. vi. p. 379.)
16. 16 Feb. 2, 13·5.
20. A star at +42° 8′.
21. A star 2′ S.
23. 16 April 4, 10·8. Var.?
24. A star 1′–2′ N.
25. 13 April 5, 12.
27. 14 March 12, 13.
31. 16 April 4. Not seen.
32. 13 April 5. Not seen.
34. A star about 20 sec. *p*.
36. 10 Feb. 11. Faint star seen. 11 March 9, 10 mag. Var.
38. A star 2′ N.
43. A star 2′ S.
44. Misplaced in chart where R.A. is 1^m too great.
45. 13 May 2, no star above 13·5. 15 March 31, not seen. 16 April 4, not seen.
49. A star 1′ N, faintly seen in H.M.
50. Several small stars near this place.
52. 15 April 29. Not seen.
54. In H.M. a star some 2′ N.
55. B.D. +42°·2856, 42·4 sec. *p*, 1′·3 S. is a wide pair 9·3, 9·3.
57. The double star Espin 636 is 4·4 sec. *f*, 7′ S. of this place.
58. A star 4′ N.
59. A star 2′ S.
60. A star 30 sec. *p*.
63. 15 Sept. 2, 12·5.
70. A star 5′ S.
73. A star 2′ N.
76. A star 2′ N.
78. 13 Nov. 7, 9·6 Argelander's scale.
79. 13 Nov. 7, 12.

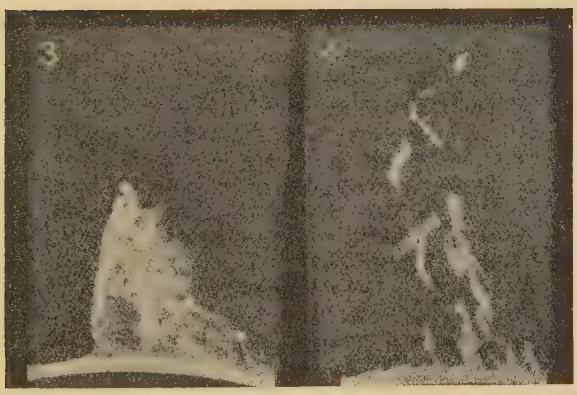
Note on the Uprush of Gases in the Solar Prominences. By the Hon. R. J. Strutt, F.R.S.

(*Communicated by the Secretaries.*)

The great velocities with which the gases shoot up radially from the level of the chromosphere have long attracted attention.



$14^h\ 4^m$ $16^h\ 27^m$



$17^h\ 0^m$ $17^h\ 40^m$

Prominence Photographs by Deslandres, 1894 May 31. The little white circle in the first figure represents the dimensions of the earth on the scale employed.

These uprushes of gas have apparently been considered analogous to the similar though infinitely less rapid movements in our own atmosphere due to convection. It has perhaps been somewhat vaguely supposed that the enormous velocity is quite in keeping with the huge scale and high temperature of solar phenomena generally.

The object of the present note is to point out what seems to be an insuperable difficulty in this view.

For purposes of discussion I take as a concrete example the photographs published by Deslandres (*Annuaire du Bureau des Longitudes,* 1907, p. 87) of a polar prominence taken on 1894 May 31. They are here reproduced for convenience of reference.

It appears from these photographs that during the interval from $14^h\ 04^m$ to $16^h\ 27^m$ the mean upward velocity of the highest part of the prominence was $4{\cdot}5 \times 10^6$ cm. per sec. But during the succeeding interval from $16^h\ 27^m$ to $17^h\ 0^m$ the velocity of the highest part had increased to $8{\cdot}34 \times 10^7$. Thus practically the whole of the latter velocity was acquired during this interval. It is important to notice that before this velocity was acquired the top of the prominence was already at a height above the general level of the chromosphere equal to seven times the earth's diameter. It appears certain therefore that this velocity was acquired by the gas not within the sun, but *after emerging into the sun's atmosphere.* This circumstance materially simplifies the discussion. If the velocity were acquired mechanically inside the sun's mass, as for instance by a huge bubble of hydrogen rising through denser material, it would be very difficult to reason quantitatively about what value the velocity might be expected to have.

I wish now to discuss whether such velocities as are observed can be attributed to convection currents, including in this term any motion of the gas under a pressure gradient, whether of thermal origin or not.

Two cases may be distinguished—convection in the form of continuous circulation, and convection in one defined upward direction, starting from rest.

First, as regards continuous circulation: this occurs under conditions analogous to those of a domestic hot-water circuit, an upward current of hot gas, and a descending current of cold gas. A circuit of this kind is analogous to a heat engine. Heat is taken in at a high temperature, and a less amount of heat is rejected at a lower temperature, the difference representing the mechanical work done in setting the gas in motion. There is no easily calculable limit to the velocity which may be acquired in a circulation of this kind. Such a limit is only set by frictional resistance encountered by the moving fluid against stationary surroundings. Apart from this, there is no reason why the velocity should not increase indefinitely, something being added to it each time the gas ascends.

Whatever the importance of motion of this kind may be at the lower levels of the sun's atmosphere, the evidence of the photographs clearly justifies us in assuming that it is not concerned in the case of the prominences. What we have to do with is *one* upward ascent, the gas starting from rest, without any kinetic energy carried on from a previous horizontal motion.

Although convection may occur in the open (as in the earth's atmosphere), yet it acts to greater advantage in setting a gas in motion if the latter is confined in a wide pipe or chimney, open at the bottom. If therefore we imagine a chimney to be placed over the prominence so as to prevent horizontal movements of the gas, we shall not make the conditions less favourable for a convective uprush than they would be in the absence of a chimney, but more so. At the same time the conditions are made definite and calculable.

Let us suppose such a chimney to be in position. Then the upward motion of the hydrogen requires a greater pressure at the bottom than at the top. As regards the top, let the most favourable supposition be made, namely, that the pressure is zero. As regards the bottom, let the pressure be p_0 dynes per sq. cm.

It might appear at first sight that the greater the value of p_0 the more rapid would be the ultimate motion; but, as is well known, for adiabatic expansion this notion is illusory.

We suppose the gas at the bottom to be at a temperature of 6600° centigrade or 6900° absolute, and to rush up into a vacuum above. What velocity will it attain in doing so?

The question may be approached conveniently by considering how much energy is available to set the gas in motion.

Suppose that 1 gram of hydrogen enters the bottom of the chimney. The work done upon it by the external atmosphere in pushing it in will be equal to

$$p_0 v_0,$$

the product of the initial pressure and volume, under the conditions prevailing outside the base of the chimney.

But we have

$$p_0 v_0 = \mathrm{R}\mathrm{T}_0,$$

when T_0 is the absolute temperature outside, and R is the gas constant.

During this initial part of the process the gas may be regarded as behaving like an incompressible fluid such as water, and it acquires velocity under the action of the superincumbent pressure, just as water does in issuing out of a small side-hole in the bottom of a vessel.

Having entered the chimney, which has a vacuum maintained at the top, the gas will rush upwards with increasing velocity. Energy of motion is derived from the expansion of the gas.

Let us suppose this expansion in the first instance to be adiabatic. Then, if p and v are the pressure and corresponding volume at any stage, and γ the ratio of specific heats,

$$pv^\gamma = p_0 v_0^\gamma = p_0 v_0 \cdot v_0^{\gamma-1} = \mathrm{R}\mathrm{T}_0 \cdot v_0^{\gamma-1}.$$

Thus

$$p = \mathrm{R}\mathrm{T}_0 v_0^{\gamma-1} \cdot v^{-\gamma}$$

and the work to be got by unlimited expansion from the initial volume v_0

$$= \int_{v_0}^{\infty} p dv = \int_{v_0}^{\infty} \mathrm{R}\mathrm{T}_0 v_0^{\gamma-1} \cdot v^{-\gamma} dv = \frac{\mathrm{R}\mathrm{T}_0}{\gamma - 1}.$$

To this must be added the work $\mathrm{R}\mathrm{T}_0$ done on the gas in entering the chimney, and we get as the total energy available to produce motion

$$\mathrm{R}\mathrm{T}_0\left(1 + \frac{1}{\gamma - 1}\right)$$

or

$$\frac{\mathrm{R}\mathrm{T}_0\gamma}{\gamma - 1}.$$

If V is the velocity generated,

$$\tfrac{1}{2}\mathrm{V}^2 = \frac{\mathrm{R}\mathrm{T}_0\gamma}{\gamma - 1}$$

$$\mathrm{V} = \sqrt{\frac{2\mathrm{R}\mathrm{T}_0\gamma}{\gamma - 1}}.$$

Taking

$$\begin{aligned} \mathrm{R} &\text{ as } 4{\cdot}158 \times 10^7 \\ \gamma &\text{ ,, } 1{\cdot}408 \\ \mathrm{T}_0 &\text{ ,, } 6{\cdot}900. \end{aligned}$$

V comes out as $1{\cdot}4 \times 10^6$ cm. per sec.

This then is the greatest velocity that could be generated by adiabatic expansion of hydrogen initially at the temperature of the sun, even if the process were carried out to an infinite volume. Yet it is only $\frac{1}{60}$th part of the velocity of the prominence. Although molecular considerations are in no way essential to the argument, it is worth while to remark on what is occurring from a molecular point of view in the case supposed. The gas enters the chimney with a certain velocity, derived from the work done upon it by the external atmosphere. It then expands, and the energy of the molecular motions, both of translation and of rotation, is converted into velocity of the gas as a whole in one direction. If the whole energy were translational as in a monatomic gas like helium, then the ultimate velocity of the gas would be equal to the molecular velocity of mean square at the initial temperature. If, as has been supposed above,* hydrogen remains diatomic at the temperature of the sun, it will acquire a somewhat larger velocity, for the initial energy of rotation is added.

It appears therefore that the greatest velocity which can possibly result from adiabatic expansion of the gas is very small compared with the velocities observed.

It may, however, be objected with reason that the expansion will not be adiabatic. Heat may be supposed to be conducted

* In taking the value $\gamma = 1{\cdot}408$.

through the walls of the imaginary chimney, and in this case the energy of molecular agitation which the gas gives up in the process of expansion will be restored to it. Let us take the case when this restoration is complete, so as to make the expansion isothermal, the gas remaining at solar temperatures throughout instead of being cooled by expansion. As before, the work done in expansion is

$$\int_{v_0}^{v_1} p dv,$$

where v_0 and v_1 are the initial and final volumes.

In the present case pv is constant, and equal to RT_0, when R is the gas constant and T_0 the absolute temperature.

Thus the work done in expansion is

$$RT_0 \int_{v_0}^{v_1} \frac{1}{v} dv$$

or

$$RT_0 \log \frac{v_2}{v_1}.$$

To this must be added as before the work RT_0 done upon the gas by the external atmosphere.

Thus if V is the ultimate velocity

$$\tfrac{1}{2}V^2 = RT_0 + RT_0 \log \frac{v_2}{v_1}$$

$$= RT_0\left(1 + \log \frac{v_2}{v_1}\right)$$

$$V = \sqrt{\left\{ 2RT_0\left(1 + \log \frac{v_2}{v_1}\right)\right\}}.$$

This expression can be made to exceed any assignable value if the expansion $\frac{v_2}{v_1}$ is made sufficiently great, and it might appear from this circumstance that there could be no difficulty in accounting for the observed velocities of the uprising gas by assuming a large enough expansion. But when the actual figures are considered the matter wears a very different aspect.

First, the photographs enable us to set a close limit on the expansion that can have actually occurred. (Compare stage 3 with stage 2.) The mean heights of the prominence in these two stages are nearly in the same ratio as the maximum heights—about 2 . 3 : 1 : The base remains about the same in each. Thus the volumes are also in the ratio 2 . 3 to 1, and if we suppose that no fresh gas has come up, this is the mean expansion.*

By assuming that all the expansion has occurred in the higher

* If fresh gas *has* come up, the mean expansion of the original gas will of course be still less.

portions of the prominence, we may get a rather larger figure. But I do not think that the most violent straining of the probabilities will allow us to assume that more than a tenfold expansion has occurred in these upper portions, particularly if the evidence of the subsequent photograph, No. 4, is taken into account.

Calculating the velocity obtainable by a tenfold isothermal expansion, we find

$$V = 1{\cdot}37 \times 10^6 \text{ cm. per sec.},$$

which is only $\frac{1}{60}$th part of the observed upward velocity of the top of the prominence.

Again, it is instructive to consider what isothermal expansion would be required if the observed velocity were to be reached in this way.

In this case we get, for a velocity of $8{\cdot}34 \times 10^7$,

$$\log_e \frac{v_1}{v_0} = 1{\cdot}21 \times 10^4,$$

which gives

$$\frac{v_1}{v_0} = 10^{5250} \text{ roughly.}$$

Such an expansion of the whole prominence, or of any appreciable fraction of it, would make it fill a space far in excess of the estimated volume of the stellar universe.

It seems therefore quite impossible to attribute the observed velocities of the hydrogen to the same kind of forces which produce the movements in our own atmosphere.

What view, then, is to be taken of these movements?

Can it be supposed that it is not the gas itself, but the *luminous condition*, which spreads upwards?

This suggestion is decisively negatived by the observations which have been made on the Doppler effect, when the motion has a component towards the observer. This effect can only occur when the material emitting centre is moving towards the observer.

Again, can we find any escape in supposing that the moving centres of emission are not hydrogen atoms, but something much lighter?

Such a suggestion would be utterly at variance with our experimental knowledge of the circumstances under which the hydrogen spectrum is emitted, and with all our theoretical notions of the origin of line spectra.

It seems much more feasible to suppose—and the notion has been contemplated before,* though I do not know that it has ever been strongly pressed—that the masses of glowing hydrogen

* See, for instance, Deslandres, *C.R.*, clv. p. 1579, 1912, "Les protubérances sont alors simplement un amas des ions lumineux qui sont rejetées au dehors par les forces *aérodynamiques ou électriques.*"

are urged upwards by electrostatic forces. There seems to be little extravagance in such an hypothesis, for Hale's observations on the Zeeman effect in sun-spots may be said to compel the assumption of strongly electrified masses of gas in rotation.

Summary.

The motion of gas in the solar prominence is discussed from a theoretical point of view, and it is concluded to be impossible to regard this motion as due to convection. The reasons for this view are—

1. The photographs show clearly that no continuous circulation of the gas is in question. We cannot therefore regard the velocity as gradually built up in successive ascents.

2. The velocities are enormously greater than can be accounted for by any adiabatic expansion at solar temperature, however great.

3. If the expansion is regarded as isothermal, the observed velocities could only be accounted for by assuming a ratio of expansion enormously exceeding what observation will admit.

Note.—Since writing the above I have found that the impossibility of explaining the motion as resulting from adiabatic expansion has previously been insisted upon by Pringsheim, who has also suggested the alternative of electrostatic force (*Physik der Sonne*, p. 225, Teubner, Leipzig, 1910). I think, however, from remarks in the subsequent literature, and from what I have been able to learn in conversation, that the above discussion may not be without usefulness in directing attention to the subject.

The Total Solar Eclipse of 1916 *February* 3.

By C. D. Perrine, Assoc. R.A.S.

This eclipse was observed at Tucacas, Venezuela, by the expedition from this observatory in charge of Third Astronomer Chaudet.

Owing largely to the financial difficulties resulting from the great European war it was not possible to send the 40-foot camera nor the 11-foot spectrographs.

Following was the programme of observations :—

A. Photographs of the corona with cameras of 11 feet and 6 feet focal length.

B. Photographs of the flash spectrum with prismatic cameras of 6 feet and 27 inches focal length.

C. Photographs of the general spectrum of the corona with a one-prism slit spectrograph.

D. Photographic determination of the total amount of light from the corona.

E. Visual observations of the times of contacts.

It was impossible to obtain fresh orthochromatic plates as well as some moderately slow plates for the extreme inner corona and photometric work.

Standard intensity squares were impressed on the long exposure for the 11-foot camera and upon the four plates for the photometer, before leaving Cordoba. They were then sealed up in tin along with the other plates for use on eclipse day, and not opened until the night before. The plates, which had been standardised before leaving Cordoba, were somewhat mottled, due, it is believed, to the exposure to the atmosphere two months before development.

The expedition left Cordoba on 1915 December 2 and arrived in Tucacas on January 14.

Although the weather for the first week was unfavourable, Mr. Chaudet succeeded in getting all of the six instruments mounted and adjusted. Assistance for taking the photographs was freely offered from among the officials and residents of Tucacas, and an efficient corps of observers was trained for a week before the eclipse.

The morning of the eclipse was unpromising, with heavy rain. Conditions improved, however, to such an extent that, during totality, the sky was covered only by haze. This interfered slightly with the coronal extensions, the general spectrum of the corona, and the photometric observations.

The entire programme was carried through, and 20 plates with 28 exposures obtained.

The foci of the 11-foot camera and of the large prismatic camera were excellent. There were some very small vibrations to the 11-foot camera which disturbed the short exposures, particularly the images of the prominences, to some extent. They were not sufficient, however, to interfere greatly with the longer exposures.

The images with the 6-foot camera and the small prismatic camera are not sharp. The latter appears not to have been in good focus, owing to lack of opportunity to focus properly the instrument before the eclipse—clouds preventing the completion of a focus plate on the stars. This instrument had not been focussed before leaving Cordoba.

Diffusion by the clouds may have caused a part of this appearance in the case of the 6-foot camera, but it is feared that some disturbance of the focus took place, although no such disturbance was noted after the eclipse, nor can any be accounted for. The focal image was examined visually shortly before totality, and was then excellent. The general features of the corona and the streamers are well enough shown for some purposes, as are also the prominences.

In the small prismatic camera negatives the general forms of the images are easily distinguishable, and many of the lines well enough separated for measurement on their sharp edges.

The negatives of the corona with both cameras show streamers to a distance of one and a half solar diameters.

The corona is of the intermediate type, resembling that of 1898 more closely than any other recent one.

Five groups of prominences are shown at the bases of the four principal wings of the corona. A series of well-marked hoods is shown over the prominences in the S.W. quadrant. One of comparatively rare, long, very slender and nearly radial streamers is shown in the N.E. quadrant. This streamer is composed of two distinct branches near the limb.

The chromospheric layer was of considerable depth.

Both flashes were obtained with the prismatic cameras; and three exposures on the corona were made (instead of a single one during all of totality) on account of the great light-gathering power of these instruments.

No trace of gaseous radiation in the corona proper has been found. The principal hydrogen lines can be seen in the prominences in the exposures at mid-totality.

The exposures with the large prismatic camera were made on flat plates. The focus from the extreme red end (λ 7300) to about H is good. Above this it is not sharp, although many lines are visible, and as the edges are generally sharp it may be possible to measure them with some accuracy owing to the large scale. The spectrum is recorded to λ 3300, where it is cut off by the edge of the field.

Bright lines are well shown to about λ 5300. Below this the sensitiveness of the plate (Lumière Violet Label) falls off considerably, and at the extreme lower end only the D_3 and H_a lines are visible. These are both strong, and the forms of some prominences are plainly discernible in them. Continuous spectrum extends to λ 7300 or below.

About 40 lines can be distinguished in the flash spectrum above λ 3600.

Probably the greatest value these observations will have is the heights of the different vapours in comparison with similar results from other eclipses.

The spectrum with the slit spectrograph is faint, and does not extend beyond the extreme inner corona, 3′ from the moon's limb on the east and not more than 1′ on the west.* The slit was placed east and west across the sun's diameter. The spectrum appears to be perfectly continuous on both sides, without a trace of either dark or bright lines, except for small bright beads on the linear continuous spectrum on the west at the H and K positions.

The photometric plates present some interesting results—chief of which appear to be that the prominences and chromospheric layer at the beginning of totality emitted more light than the corona proper. It may be possible to determine the total amount of light from some of the groups of prominences separately, as distinct rings can with some difficulty be traced on the negative taken near the end of totality.

* The exposure was stopped intentionally 20 seconds before the end of totality.

The second and third contacts were well observed by Mr. Chaudet as follows:—

1916 *February* 3, *Tucacas Mean Time.*

	Observed.				Computed.			
	h	m	s		h	m	s	
Beginning of totality	11	23	21	a.m.	11	23	21	a.m.
End „ „	11	25	51	„	11	25	53	„
Duration		2	30			2	32	

These agree very closely with the times obtained by the party from the Caracas Observatory. The computed times are given for comparison, the data being taken from the American Ephemeris and the correction of -10^s given by Ross in *Astronomical Journal*, No. 681, having been applied.

It will be seen that the beginning of totality agreed within the second, and the end differed but 2^s from these predictions.

The position of Tucacas obtained by a commission from the Caracas Observatory is as follows:—

Latitude . . . 10° 47′·5 North.
Longitude . . 68 19·2 West of Greenwich.

Observatorio Nacional Argentino,
Cordoba:
1916 *June* 26.

The Rotation Period of the Great S. Tropical Disturbance on Jupiter. By Frank Sargent.

The Great S. Tropical Disturbance on the planet Jupiter has shown a curious variation in its periods of rotation during recent years.

In a general way these have been lengthening. The object being subject to considerable changes in length, it naturally follows that the periods of its ends will likewise be subject to corresponding fluctuations. The purpose of this note is to direct attention to a peculiar oscillation in the values for these ends.

A very usual method of tabulating the various periods of Jovian features is to evaluate them as from "opposition" to "opposition." But observations are only possible for a portion of the time, and during any such observing season the rotation periods derived may, and frequently do, vary from those derived from "opposition" to "opposition."

By using positions observed at the close of an "opposition," and others obtained at the beginning of the next, it is also possible to derive values for the time of the planet's invisibility.

Recently it has been noted that the rotation periods derived for the ends of the S. Tropical Disturbance have been longer during the time of its visibility than during the time when Jupiter has been lost in the Sun's rays.

This has been consistently so during the last three years (beyond which my personal records do not go with sufficient amplitude to justify their use respecting this matter), and recent observations indicate that the peculiarity is being maintained during the present opposition.

During this time the S. Tropical Disturbance has varied considerably in length, but the changes have not succeeded in breaking down the sequence of these up-and-down movements in the rotation periods of its ends.

In support of the foregoing the following data are submitted, consisting of certain dates, with corresponding positions of longitude for the ends of the S. Tropical Disturbance believed to be well established, and the rotation periods derived for the intervals between succeeding dates.

The terms "opposition" and "conjunction" are used to cover the intervals during which these events take place.

S. T. D. Rotation Periods.

Preceding End:—

From	To	Period. h m s	☿ in
1913 June 14 λ_2=210	1913 Sept. 26 λ_2=184	9 55 29·3	Opposition.
1913 Sept. 26 ,, 184	1914 July 11 ,, 80·7	,, 25·9	Conjunction.
1914 July 11 ,, 80·7	1914 Nov. 7 ,, 47·3	,, 29·1	Opposition.
1914 Nov. 7 ,, 47·3	1915 June 8 ,, 339·6	,, 27·6	Conjunction.
1915 June 8 ,, 339·6	1916 Feb. 5 ,, 268·8	,, 28·6	Opposition.
1916 Feb. 5 ,, 268·8	1916 July 19 ,, 206·4	,, 25·2	Conjunction.
1916 July 19 ,, 206·4	1916 Oct. 16 ,, 178·5	,, 27·7	Opposition.

Following End:—

1913 June 27 λ_2=346	1913 Sept. 15 λ_2=320	9 55 27·6	Opposition.
1913 Sept. 15 ,, 320			*
1914 Aug. 5 ,, 185·1	1914 Nov. 17 ,, 153·5	,, 28·1	Opposition.
1914 Nov. 17 ,, 153·5	1915 Aug. 13 ,, 52·7	,, 25·4	Conjunction.
1915 Aug. 13 ,, 52·7	1916 Jan. 24 ,, 13·7	,, 30·9	Opposition.
1916 Jan. 24 ,, 13·7	1916 July 22 ,, 317·6	,, 27·8	Conjunction.
1916 July 22 ,, 317·6	1916 Oct. 15 ,, 298·3	,, 31·3	Opposition.

It will be noted that the variations are of such magnitude that observational errors sufficient to remove them seem impossible.

* (When Jupiter came under observation in May 1914 the F.E. of the S. Tropical Disturbance was involved with the Red Spot Hollow, with a result that it was not possible to observe its position accurately, therefore a rotation period for the preceding months cannot be given. On Aug. 5 an excellent observation re-established the position.)

Respecting this I desire to point out that observations of the "Red Spot Hollow" during the same time show no such oscillation, and a prominent object on the S. Temperate Belt recently re-observed from last "opposition" yields a rotation period for the interval precisely the same as that derived during 1915.

These consistent results from features contiguous to the zone in which the S. Tropical Disturbance is situated seem to negative the possibility of observational error sufficient to account for the curious fluctuations recorded.

The search for a physical explanation has not been successful. The relative positions of the Sun, Earth, Jupiter system seem clearly involved.

Gravitational effects may be immediately dismissed, in the first place, because of their minuteness as far as the Earth is concerned; and secondly, because in view of their proximity the

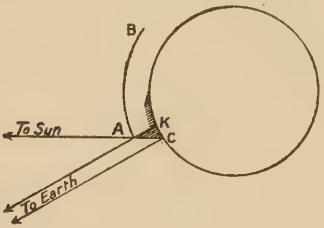


FIG. 1.

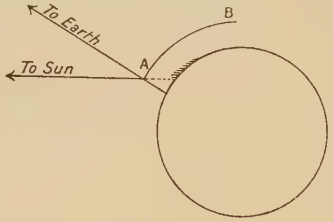


FIG. 2.

satellites of Jupiter are in this direction a much more potent factor.

The phenomenon has been considered on the assumption that the S. Tropical Disturbance is of a cloud formation, capable of casting a shadow on to the light zone which it overlays.

During the months previous to the date of "opposition" the shadow cast by the P.E. of the S. Tropical Disturbance will be in view of an observer on the earth, whilst after "opposition" the shadow of the F.E. will be in a position to be seen. This being so, it is highly probable that the S. Tropical Disturbance being itself a dark feature, will merge itself in its shadow as seen from the Earth, and when the shadow is in a position to be seen, it may be timed by the observer, instead of the actual material of the S. Tropical Disturbance.

The effect of this may be inferred after consideration of figs. 1 and 2.

In fig. 1 the positions shown are as at "quadrature" preceding "opposition."

Jupiter is represented by a circle, the S. Tropical Disturbance by the arc AB: A being the P.E. The shadow CK will be seen from the Earth, resulting in the time being taken early, the actual amount being taken from the "conjunction" rotation periods, and eventually distributed over the "opposition" periods,

or more correctly, over the periods from the time of the observation until the time of true "opposition," when the shadow will cease to be visible from the Earth.

From the date of "opposition" onwards the shadow will not be visible from the Earth, being overlaid by the S. Tropical Disturbance itself, as may be seen from fig. 2.

For the F.E. the process will be reversed, the shadow being invisible until after the date of opposition, when it will appear as an extension of the S. Tropical Disturbance, resulting in late timing of transits, and a corresponding lengthening of the rotation periods.

As a question of theory this may be accepted, but as a matter of actual observation the magnitude of the shadow effect is very small indeed, probably not large enough to influence observations at all, unless we assume a height for the alleged "cloud" for which we have no warrant. The suggestion is of interest only because it agrees in kind and direction with the motion necessary to explain the observations.

Further investigation seems desirable, is in fact proceeding, and, aided by earlier observations kindly supplied by the Rev. T. E. R. Phillips, M.A., and Messrs. W. F. Denning and Stanley Williams, it is hoped to carry the inquiry back into the earlier history of the object under consideration.

16 *Cottrell Road, Eastville,*
Bristol:
1916 *November* 1.

The Meteor System of Pons-Winnecke's Comet.

By Charles P. Olivier, M.A., Ph.D.

Before discussing the question of the connection between the 1916 May-June meteors and the Pons-Winnecke Comet, it is of interest to review briefly the history of this body.

It was first discovered by Pons on 1819 June 12, and though its period was computed at that time to be only 5·62 years, it escaped observation until 1858 March 8, when it was rediscovered by Winnecke at Bonn. It has since reappeared in 1869, 1875, 1886, 1892, 1898, 1909, and 1915.

This comet belongs to Jupiter's family, and at different times in its history has approached very near that planet. The strong resulting perturbations have greatly changed its orbit. Fortunately, the progression of the changes is very easy to follow, thanks to several papers on the subject. It will be sufficient to mention an article by Winnecke in *A.N.*, No. 1160, which gives the 1858 orbit; one by von Haerdtl in *A.N.*, No. 2873, which gives orbits for 1858, 1869, 1875, and 1886; and, lastly, one by Hillebrand in *A.N.*, No. 4330, for the 1909 return. Unfortunately, no orbit

seems to have been computed from the few observations made in 1915. On this last return it was discovered in April, when of the 16th magnitude, and at its closest approach to the earth was more than a unit distant. At its brightest it was of the 12th magnitude and 20″ in diameter, as observed at the Union Observatory, South Africa.

The three orbits for 1819, 1858, and 1909 follow, so that the changes in the elements may be readily seen:

Year.	log *a*.	log *e*.	log *q*.	*i*.	π.	Ω.	U.
				° ′	° ′	° ′	*y*
1819	0·4997	9·8781	9·8885	10 43	274 41	113 11	5·62
1858	0·4965	9·8778	9·8859	10 48	275 39	113 32	5·56
1909	0·5135	9·8462	9·9879	18 17	271 37	99 21	5·892

If all the orbits between 1858 and 1909 were to be included in the table, they would show progressive changes in all the elements from 1858 on. However, as these two mentioned show their directions clearly, it is thought unnecessary to copy in the others.

The element of greatest interest to us is the perihelion distance. It will be seen at once that this has increased from 0·77 in 1858 to 0·97 in 1909, and doubtless has continued to increase since. This increase in distance has made possible our meeting with the meteors which belong to the comet's system.

Some rough calculations show that on June 28·5 the comet's (1909) orbit was about three million miles from the earth, while on May 27 it was nearly five times as distant. That we should meet outlying meteors even at this great distance from the parent comet's orbit has already been observed in the cases of the Perseids and the Eta Aquarids (the Halley Comet meteors).

The observations of these meteors for testing their connection with the Pons-Winnecke Comet are quite numerous, thanks to the energy of some of the observers who are members of the American Meteor Society. Altogether, in the interval 1916 May 20 to July 10, which seems about the extreme limits advisable to use in the discussion, five observers, working in all 47 nights, sent in 1219 observations of meteors. Also in *Nature* of July 10 two English observers, working on June 28, reported, one seeing a large number of meteors, the other finding a radiant between ζ and η Ursæ Majoris, with a rate of 100 ± meteors per hour. Later, in *The Observatory* for August, Mr. W. F. Denning gives his own observations, with a rate of 32 per hour and the radiant being diffused, having two principal centres. In *Nature* of July 17 Mr. Denning says that the English observations show some resemblance in their elements to those of Pons-Winnecke's Comet. Absolutely no mention of this is made in his more complete article in *The Observatory* of August. In the September number of that journal, however, he says that there is a significant resemblance. All English observers reported that meteors were visible in extraordinary numbers on June 28. Unfortunately, only one of the

American observers, Mr. Donald Brooks of Washington, D.C., worked on June 28. His rate was only 5·5 per hour. This need, however, excite no surprise, because it has long been known that the richest portions of meteor streams, even the Leonids, are traversed in a comparatively few hours, hence the six hours which elapsed between the English and American observations were quite sufficient to allow the earth to pass out of the dense portion.

From our data seven radiants are clearly shown on the maps of Mr. John Koep and Philip Trudelle, of Chippewa Falls, Wis., between May 21 and June 4 inclusive, which certainly belong to this system, and one radiant observed by Mr. Raymond Lambert of Newark, N.J., on July 3, which almost certainly does also.

Using the eight radiants determined in America, the radiant at $\frac{1}{2}(\zeta+\eta)$ Ursæ Majoris by the observer at Birmingham, and Mr. W. F. Denning's two principal centres, orbits have been computed for all eleven positions. They were first computed on the usual assumption of parabolic velocity. When this had proved that the meteors followed orbits similar to that of the comet, elliptical orbits were next computed for each, assuming that their major axes were equal to that of the comet. This produced so marked an improvement in the agreement of the elements with those of the comet that no reasonable doubt could remain, and the connection may be considered fully proved.

The two tables of elements follow :—

Parabolic Elements.

Date. G.M.T. 1916.	No.	α.	δ.	ι.	log q.	ω.	π.	Ω.	Observer.	Nos.	Notes.
May 21·6	1	224°·5	+25°·3	20°·2	9·954	219°·0	279°·7	60°·7	J.K.	8	4 on May 19, 4 on May 23.
May 26·68	2	230°·3	+27°·4	22°·4	9·958	217°·9	283°·4	65°·5	P.T.	7+	
May 27·25	3	231°·0	+27°·5	22°·5	9·958	218°·1	284°·2	66°·1	J.K.	7	4 on May 26, 3 on May 27.
May 27·67	4	232°·1	+26°·8	22°·8	9·954	218°·9	285°·4	66°·5	P.T.	4+	
May 30·14	5	232°·7	+28	22°·1	9·978	209°·0	277°·9	68°·8	J.K.	15	8 on May 29, 7 on May 30.
June 3·7	6	234°·4	+27°·5	22°·2	9·965	215°·1	288°·4	73°·2	J.K.	16	June 2, 3, 4.
June 4·68	7	235°·8	+25°·6	22°·0	9·961	216°·9	291°·0	74°·2	P.T.	9	
June 28·5	8	203	+53	20°·9	0·005	171°·4	268°·3	96°·9	B.	100∓	
June 28·5	9	231	+54	27°·3	0·007	182°·9	279°·8	96°·9	W.F.D.	?	69 seen.
June 28·5	10	223	+41	19°·7	0·006	186°·8	283°·7	96°·9	W.F.D.	?	
July 3·67	11	206°·7	+61°·2	26°·0	0·001	165°·8	267°·6	101°·8	R.L.	5	

Note.—The observers were :—J.K. = John Koep.
P.T. = Philip Trudelle.
B. = Observer at Birmingham.
W.F.D. = W. F. Denning.
R.L. = Raymond Lambert.

Elliptical Elements.

No.	log a.	log e.	log q.	ι.	π.	ω.	Ω.
1	0·514	9·857	9·962	16°·7	280°·1	219°·4	60°·7
2	,,	9·865	9·948	18·6	287·6	222·1	65·5
3	,,	9·854	9·970	17·6	282·6	216·1	66·1
4	,,	9·856	9·964	19·0	286·6	220·1	66·5
5	,,	9·848	9·983	16·6	278·0	209·2	68·8
6	,,	9·852	9·974	17·6	287·6	214·3	73·2
7	,,	9·855	9·967	18·2	291·6	217·4	74·2
8	,,	9·839	0·005	17·1	2·683	171·4	96·9
9	,,	9·840	0·002	22·9	303·0	206·3	96·9
10	,,	9·839	0·005	16·1	285·2	188·3	96·9
11	,,	9·857	9·962	22·6	263·7	161·8	101·8
Comet (1909)	0·514	9·846	9·988	18·3	271·6	172·3	99·3
Comet (1858)	0·496	9·878	9·886	10·8	275·6	162·1	113·5

It will be seen that, contrary to our rule, the mean positions from more than one night's work have been combined in Nos. 1, 3, 5, and 6. This was due to the fact that Mr. Koep used four maps in all, and it was impossible to detect appreciable deviations in radiant points for the meteors shown on each map when taken for each night separately, and then as combined into the adopted means. It goes without saying that the remarkably exact progression shown in the first seven positions is partly due to good fortune. Nevertheless, all the radiants were well determined and quite sharply defined on the maps.

If it be objected that the node positions differ considerably for the earlier dates, it must be remembered that the Ω for a meteor stream is simply the longitude of the earth on that date, and such differences only inform us that the meteors are met at a considerable distance from the parent comet's orbit. Hence rather considerable disagreements in Ω may mean little or nothing provided the a, e, q, and π are accordant. Again, we must remember the comet passed perihelion in 1915 September, and therefore was an enormous distance ahead of these meteors. Therefore the perturbations to which it and these groups had been of late years subjected were by no means the same, and some differences of elements are to be fully anticipated.

If the perihelion distance of the comet's orbit, and hence inferentially those of the meteors, continues to increase as rapidly as in recent years, it seems that this interesting system of meteors may be carried beyond the earth's orbit, and hence can be seen for only a few years more at most. It was never observed before 1916, so far as I have been able to find, and, as has been explained,

unless the changes in the elements of the orbits shortly reverse their direction, it cannot long be visible to us. It is therefore of very real importance to make every effort to secure observations in 1917 May and June, and probably for the next few years. It is believed that this exact cycle of changes just described offers a new case in meteoric-cometary astronomy, paralleled partially only by that of Biela's Comet.

The way in which the connection was detected here was that partial calculations, in June, showed that there was strong probability of the meteors belonging to the Pons-Winnecke Comet. Not until further data came in was I willing to publish anything on the subject, however, though the results were shown to the members of our observatory staff at once. Other data coming later in June made the connection certain, but not until the report from England was seen in July, giving a new and distant radiant by the observer at Birmingham, for which an orbit was calculated at once, did I think it necessary to publish the conclusion immediately, since I desired first to read a paper on the subject before the American Astronomical Society at its meeting late in August. The preliminary results were then sent to Professor E. C. Pickering on August 3, and kindly announced by him in Harvard Circular No. 614. About four days after sending this on a later copy of *Nature* arrived here from England, stating that Mr. Denning had arrived at possibly similar conclusions, though no orbits were given.

While of course the working up of the American observations and all the calculations are my own, yet I wish to give all possible credit to the two young men mentioned, through whose observations, made with the greatest energy and enthusiasm, this proof was first made possible. It is of interest to say that during the first seven months of 1916 Mr. Koep sent in 1429 observations, and Mr. Trudelle 908. This in itself is quite sufficient guarantee of their ability.

Leander McCormick Observatory,
University of Virginia,
Virginia, U.S.A.:
1916 *Sept.* 14.

Observations of the Perseids on 1916 *August* 10 *and* 11, *made at the temporary University Observatory, Rostow-on-Don.* By Professor S. D. Tscherny.

August 10.

No.	Rostow-on-Don Mean Time of Appearance.	The Point of Appearance. $\alpha_{1855\cdot0}$	$\delta_{1855\cdot0}$	The Point of Disappearance. $\alpha_{1855\cdot0}$	$\delta_{1855\cdot0}$	Remarks.
	h m s	°	°	°	°	
1	13 26 2	44·0	+53·0	73·0	+57·0	White.
2	13 38 7	44·7	+40·3	32·1	+37·6	White; streak.
3	13 40 32	28·7	+41·5	11·0	+42·7	White; streak.
4	13 47 37	81·1	+43·8	80·1	+39·0	White.
5	13 52	72·9	+43·7	77·3	+40·3	White.
6	13 59 13	91·6	+59·0	107·7	+55·4	White.
7	14 10 57	87·2	+44·8	100·0	+53·6	White; trained.
8	14 18 44	57·0	+39·5	60·0	+39·8	White.
9	14 24 45	87·4	+37·2	91·6	+33·0	
10	14 33 44	12·2	+59·9	341·4	+65·4	
11	14 38 13	58·9	+49·9	62·0	+47·0	
12	14 50 22	72·6	+60·2	91·0	+57·4	
13	14 56 39	60·0	+62·0	68·2	+65·8	
14	15 8 46	80·8	+32·1	80·0	+28·5	White; streak.
15	15 30 18	36·4	+56·1	39·2	+49·7	White.
16	15 34 59	68·4	+42·8	74·2	+41·5	Meteor of second mag.; white.

August 11.

17	12 52 46	0·6	+58·4	347·2	+53·8	Meteor of second mag.; white; streak.
18	12 55 7	5·9	+62·5	357·2	+67·1	Meteor of second mag.; white; streak.
19	13 3 55	226·8	+83·3	222·5	+74·8	Meteor of first mag.; white; streak.
20	13 22 43	15·1	+54·0	5·8	+46·4	
21	13 48 57	44·8	+49·0	57·6	+56·0	Meteor of third mag.; white; streak.
22	13 52 18	41·0	+52·2	38·9	+42·9	Meteor of fourth mag.; white; streak.
23	13 57 11	43·8	+52·9	44·0	+51·6	White.
24	14 13 49	53·5	+21·6	50·9	+16·5	
25	14 44 53	48·5	+49·3	48·7	+46·1	
26	14 50 49	49·7	+50·3	67·0	+58·3	White; streak.
27	15 2 10	51·0	+46·9	52·4	+43·5	White.
28	15 21 38	43·8	+56·1	46·0	+52·9	
29	15 27 12	54·0	+41·5	54·0	+36·1	

I have drawn the apparent paths of preceding meteors on the meteor-chart in gnomonical projection, and by the method of least squares I have obtained the three following apparent radiants:—

(1) The radiant of the meteors Nos. 1, 5, 6, 9, 12, 13, 15:

$$\alpha_{1855\cdot0} = 42^{\circ}\cdot2, \quad \delta_{1855\cdot0} = +55^{\circ}\cdot6,$$
$$\alpha_{1916\cdot0} = 43^{\circ}\cdot3, \quad \delta_{1916\cdot0} = +55^{\circ}\cdot9.$$

(2) The radiant of the meteors Nos. 2, 3, 8, 10, 11, 16:

$$\alpha_{1855\cdot0} = 61^{\circ}\cdot8, \quad \delta_{1855\cdot0} = +44^{\circ}\cdot3,$$
$$\alpha_{1916\cdot0} = 62^{\circ}\cdot9, \quad \delta_{1916\cdot0} = +44^{\circ}\cdot5.$$

(3) The radiant of the meteors Nos. 21, 22, 23, 25, 26, 27, 28, 29:

$$\alpha_{1855\cdot0} = 49^{\circ}\cdot0, \quad \delta_{1855\cdot0} = +50^{\circ}\cdot3,$$
$$\alpha_{1916\cdot0} = 50^{\circ}\cdot1, \quad \delta_{1916\cdot0} = +50^{\circ}\cdot5.$$

The meteors Nos. 4, 7, 14, 17, 18, 19, 20, 24 have not a definite radiant.

Near the constellation of *Perseus* were seen on August 10 forty-three meteors from $11^{h}\ 33^{m}$ till $15^{h}\ 43^{m}$, and on August 11 seventy-one meteors from $12^{h}\ 43^{m}$ till $15^{h}\ 47^{m}$ Rostow-on-Don mean time. The moon was near full, and set on August 10 about 13^{h} and on August 11 about $14^{h}\ 30^{m}$ Rostow-on-Don mean time.

Position of the temporary University Observatory (the centre of the platform on sixth floor of the University building): longitude, $2^{h}\ 38^{m}\ 49^{s}\cdot52$ E. (Greenwich); latitude, 47° 13′ 9″·4 N.

University Observatory, Rostow-on-Don:
1916 *August* 27.

On a Graphical Construction for obtaining the Wave-lengths in Prismatic Spectra. By F. Henroteau, D. ès Sc.

It has been established that in the case of a prismatic dispersion the Hartmann formula

$$\lambda = \lambda_0 + \frac{c}{R_0 - R} \quad . \quad . \quad . \quad . \quad (1)$$

(where λ is the wave-length, R the micrometer reading, and λ_0, c, and R_0 computed constants) gives good results for the values of λ. As a rule, then, a small correction only has to be used for obtaining the real value of λ. In astrophysical observatories this correction is obtained from a curve in which the abscissæ are the wave-lengths and the ordinates are the corrections, several corrections being deduced from known lines. The computation of λ by the

Hartmann formula, although simple, is rather a laborious operation, and a graphical method might be advantageous.

The graphical method given below is very simple, each wave-length being obtained by the tracing of only one straight line.

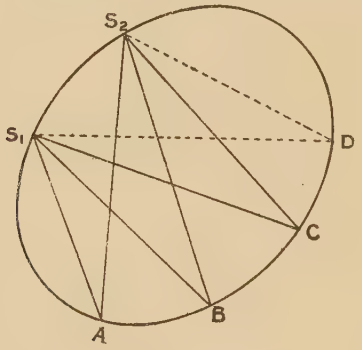


FIG. 1.

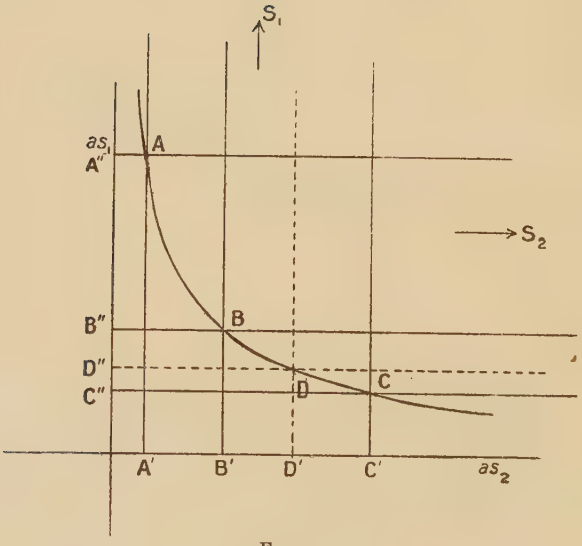


FIG. 2.

I shall first explain the theory of this construction. The Hartmann formula may be written

$$(\lambda - \lambda_0)(R - R_0) = c \quad . \quad . \quad . \quad . \quad (2)$$

This is the equation of a rectangular hyperbola, the axes of co-ordinates being parallel to its asymptotes. But if on any conic section (fig. 1) we take five fixed points S_1 S_2 A B C, and a

moving point D, we know that the pencil S_1ABC is projective with the pencil S_2ABC. In other words, the cross ratio or double ratio of those four rays is the same for each pencil.

Let us now consider a branch of the above rectangular hyperbola and its two asymptotes as_1 and as_2. Let us take ABC anywhere on this branch, and let S_1 and S_2 be the points at infinity on the same branch (fig. 2).

S_1ABC is then a pencil of rays parallel to as_1, and S_2ABC a pencil parallel to as_2.

These two pencils are projective; hence the range of points A″ B″ C″ D″ is projective with the range of points A′ B′ C′ D′.

But A′B′ and A′C′ represent certain increments of the micrometer

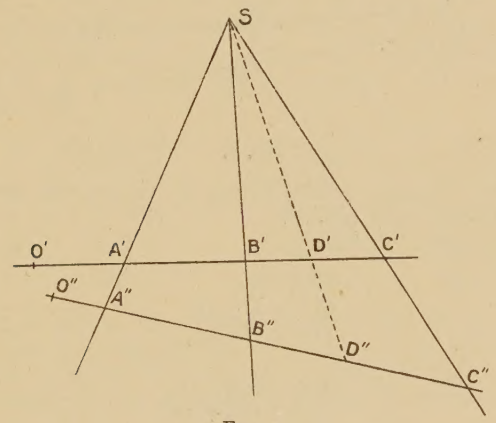


Fig. 3.

reading, and A″B″, A″C″ the corresponding known increments for the wave-length. We may then, knowing any increment A′D′ of the micrometer reading, obtain the corresponding increment of the wave-length; we have simply to put these two ranges of points in perspective. Hence we shall have the following simple construction (fig. 3).

Suppose we know three micrometer readings R_1 R_2 R_3 and the three corresponding wave-lengths λ_1 λ_2 λ_3.

Let us take a graduated scale (or piece of paper) where

$$O'A' = R_1$$
$$O'B' = R_2$$
$$O'C' = R_3$$

and let us draw that part of the scale in A′B′C′. Having taken a point S outside A′B′, we join SA′, SB′, SC′. Taking then another graduated scale, where $O''A'' = \lambda_1$, $O''B'' = \lambda_2$, $O''C'' = \lambda_3$, we fit it so that A″, B″, and C″ be each on the corresponding rays joining S to A′, B′, and C′.

O″D″ will be the wave-length corresponding to the micrometer reading O′D′ = R.

The construction becomes easier still if we have to deduce a

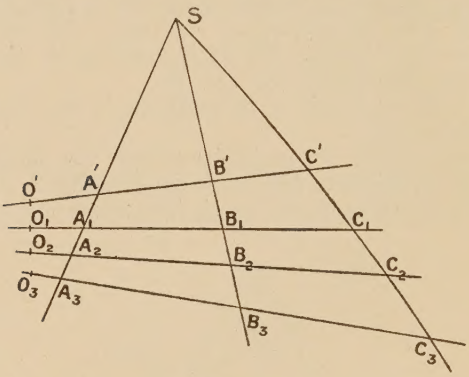


FIG. 4.

table of wave-lengths corresponding to micrometer readings increasing systematically (fig. 4).

For instance, if we take

$$O'A' = R$$
$$O'B' = R + 10$$
$$O'C' = R + 20,$$

we have

$$O_1A_1 = f(R)$$
$$O_1B_1 = f(R + 10)$$
$$O_1C_1 = f(R + 20).$$

But we may take also

$$O'A' = R + 30$$
$$O'B' = R + 40$$
$$O'C' = R + 50$$

and have the corresponding O_2A_2, and so on.

So that, if A′C′ is divided, say in twenty parts, each of the twenty-one rays drawn from S will serve more than once in dividing the series of lines A_1C_1, A_2C_2, A_3C_3. . . .

This construction has given good results. It has the advantage of being mathematically exact (as far as the Hartmann formula is concerned) and also of occupying a very small space. This space may be reduced proportionately to the number of lines A_1C_1, A_2C_2 . . . to be considered.

To give an idea of the accuracy of the construction, I have applied this method to the reduction of part of a photograph taken at Ann Arbor.

In the Hartmann formula the constants are

$$\lambda_0 = 2213{\cdot}640, \qquad c = 223802{\cdot}23, \qquad R_0 = 185{\cdot}336.$$

Having the following micrometer readings with their corresponding wave-lengths

R.	λ.
40·755	3761·577
50·668	3875·521
59·963	3998·731

I desired the wave-lengths corresponding to each integral value of R, from 40 to 60.

I took then (see fig. 3) on A′C′, each value $= \frac{1}{2}$ inch (A′C′ = 10 inches), and the point S at a distance of 7 inches from A′C′ on a perpendicular to A′C′ at the point marked 50.

I took A″C″ so that each tenth of an inch would correspond to an Ångström, and made the construction as indicated above.

In the following table the wave-lengths corresponding to the values of R, 40, 41, 48, 52 will give an idea of the accuracy of the results :—

R.	Measured Diff. in Ångströms.	Wave-lengths from the Drawing.	Computed Wave-lengths.
40	− 8·00	3753·577	3753·535
41	+ 2·65	3764·227	3764·204
48	+ 81·65	3843·227	3843·236
52	+130·60	3892·127	3892·123

We thus obtain an accuracy of about $\frac{1}{20}$ of an Ångström. A yet greater accuracy (of $\frac{1}{100}$ or $\frac{1}{1000}$ A.) may be obtained by taking more values of R and λ, and applying the fig. 4.

The construction is identical for finding R, knowing λ.

P.S.—The construction has been tried at the Yerkes Observatory. An accuracy greater than that mentioned above has been obtained by Mr. Hubble. When applied to the reduction of spectra obtained with the three-prism spectrograph, it gives perfect results up to the thousandth of an Ångström.

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1916 *October* 8.